

Optimal Magnet Pole Design of IPM Synchronous Motor for Cogging Torque Reduction

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Abstract

This paper deals with the optimal magnet pole design on both cogging torque reduction and running torque to improve an interior permanent magnet synchronous motor (IPMSM) responding surface methodology (RSM). The RSM has been achieved to use the experimental design method in combination with finite element method (FEM) and adaptive genetic algorithm (AGA). The feasibility of using RSM with the FEM and AGA in practical engineering problems is investigated with computational examples. The validity of this method is verified by comparing optimized model with original model. The simulation results of a 6-pole 36-slot IPMSM show the effectiveness of the proposed method.

Keywords: response surface methodology, genetic algorithms, finite element method, cogging torque, interior permanent magnetism synchronous motor

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1. Introduction

In recent years, the permanent magnet synchronous motor (IPMSM) has become popular because of its high power density, high efficiency, and excellent acceleration characteristics. Even though IPMSM has such advantages, it produces significantly large cogging torque due to the same length of mechanical and effective magnetic air gaps. Therefore, the reduction of cogging torque that may cause vibration and acoustic noises becomes an increasingly critical issue in IPM motor [1][2]. The cogging torque is introduced by the interaction between the magnet poles and slots. Specially, while being operated at low speed, the cogging torque will result in vibration, noise, and then reduce the performance of IPMSM.

The conventional cogging torque reduction methods include main shape of stator tooth tip, slot opening width, air gap length, pole-arc to pole-pitch ratio of magnet, magnet configuration and magnetization distribution, and skewing of either stator teeth their recent research find, magnet poles [3][4]. Kang et al. [5], in a recent result of research, have presented a novel kind of rotor core design with several notches in IPMSM to reduce the cogging torque. However, this method is considered a notch as the only design parameter. This paper deals with

the optimal magnet pole design on cogging torque reduction for an IPMSM using response surface methodology (RSM). The objective is to study how the cogging torque can be reduced and running torque performance can be improved. In addition to the notches on rotor pole face, the magnet pole position and the angle of pole-arc are selected as design parameters, which influence the cogging torque mainly. RSM is widely used in industry because of its simplicity and practicality [6][7]. The RSM has been achieved by using the experimental design method in combination of FEM and well-adapted to make analytical model for a complex problem of considering a lot of interaction of design variables [8][9]. Finite element method (FEM) is first used as an analysis tool to obtain experimental data. Then, the polynomial approximation system, which describes the relationship between performance and design variables, is derived by using the response surface methodology. The parametric regression model can be used to determine the optimal design parameters by an adaptive genetic algorithm (AGA).

The remainder of this paper is organized as follows: Section 2 shows the ipmsm original design and problem definition. Section 3 shows the response surface method using central composite design. Section 4 describes experiment and analysis. Section 5 shows the optimization methods and analysis. Finally, Section 6 presents the conclusions of this paper.

2. Ipmsm Original Design and Problem Definition

In this paper, the motor investigated is a 6-pole 36-slot interior permanent magnet synchronous motor. The original design is shown in Table 1. A two-dimensional (2-D) finite-element (FE) model has been developed from the magnetic flux calculation. Figure 1 shows the flux distribution of the original design IPMSM.

The magnet pole position, the angle of pole-arc and the notches on rotor pole face are chosen as the control factors. Figure 2 shows the definition of the three control factors T , θ_p , and θ_r . The boundary α is the limit value for the motor structure. T is the pole shift in terms of which the negative represents move toward axis, and the positive represents move toward stator. θ_p is the tangent angle of pole-arc. θ_r is the angle of notch in terms of which being in the negative represents the notch changes large, and being in the positive represents the notch changes small. The variable levels are defined as in Table 2.

Table 1. Specifications and dimensions of original design

| | |
|-----------------------------|-------------|
| Phase | 3 Φ AC |
| P | 8 HP |
| V_{L-L} | 220 V |
| Speed | 1200 rpm |
| Slot | 36 |
| Pole | 6 |
| Air gap length | 1.75 mm |
| Stator outer diameter | 160 mm |
| Iron core accumulates thick | 80 mm |
| Magnetic pole length | 4.7 mm |
| Magnet material | NdFeB N-30 |
| B_r | 1.05 Tesla |
| H_c | -779443 A/m |

3. Respond Surface Method by Using Central Composite Design

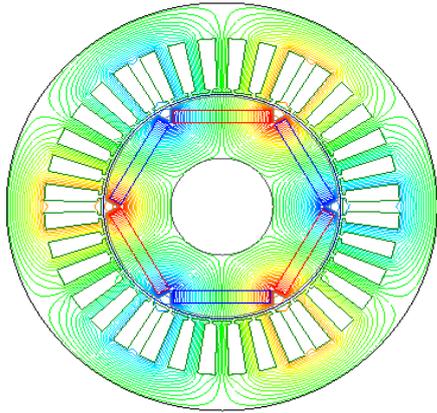


Figure 1: Flux distribution of 6-pole 36-slot IPMSM for original design

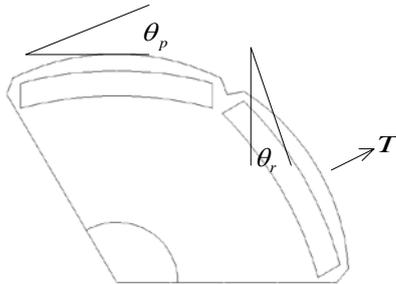


Figure 2: Definition of the three control factors

Table 2. Control factor parameters

| Control factors | $-\alpha$ | -1 | 0 | 1 | α |
|---------------------------------------|-----------|----|----|----|----------|
| Pole shift T (0.1mm) | -15 | -9 | 0 | 9 | 15 |
| Angle of pole-arc θ_p (degree) | 0 | 10 | 25 | 40 | 50 |
| Angle of notch θ_r (degree) | -10 | -2 | 10 | 22 | 30 |

The types of RSM usually include first-order factorial RSM, second-order factorial RSM, and double response surface methodology which are applied to the robust design. The goal of first-order factorial RSM design is to get model with less experiment times to approximate real system. If the system is too complex, the precise model will be unable to obtain. This paper uses second-order factorial RSM to obtain approximate real system model. The central composite design (CCD) is a popular method that can be applied to construct the second-order factorial model. A common design of experiments (DOE) is the rotatable Central composite design (CCD) that consists of 18 experiments as shown in Figure 3. These experiments include:

- 1). Eight cube points at $(\pm 1, \pm 1, \pm 1)$.
- 2). Six star points at $(\pm 1.682, 0, 0)$, $(0, \pm 1.682, 0)$, and $(0, 0, \pm 1.682)$.
- 3). Four points at the origin $(0, 0, 0)$.

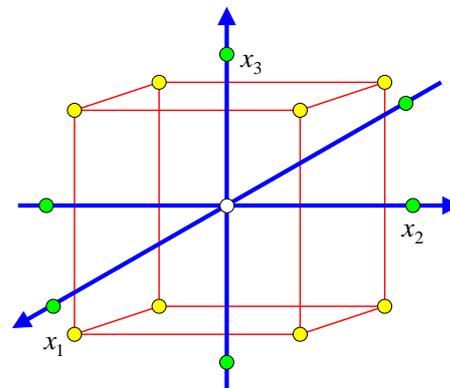


Figure 3: Central composite design

Following the program of experimentation, the data are analyzed and the results of the analysis are displayed in table. The table is called an analysis-of-variance (ANOVA) table as shown in Table 3. In Table 3, n is the total number of experiments and p is the number of parameters in fitted model. Residual degree of freedom is total degree of freedom subtracted return degree of freedom. The total variation in a set of data is called the total sum of squares (SST)

$$SST = \sum_{i=1}^n (y_i - \bar{y})^2 \quad (1)$$

where y_i is the deviations of the observed data. \bar{y} is y_i 's average value. The SST can be partitioned into two parts which are the sum of squares due to regression and the sum of squares unaccounted by the fitted model. The sum of squares unaccounted by the fitted model (SSE) is

$$SSE = \sum_{i=1}^n (y_i - \hat{y}_i)^2 \quad (2)$$

where \hat{y} is the predictability of the observed data. The formula for calculating the sum of squares due to regression (SSR) is

$$SSR = \sum_{i=1}^n (\hat{y}_i - \bar{y})^2 \quad (3)$$

Mean Square Regression

$$MSR = SSR / (p - 1) \quad (4)$$

Mean Square Residual

$$MSE = SSE / (n - p) \quad (5)$$

The F-statistic discriminates the model that obtained whether it précises or not, and the value F more greatly expressed the obtained model accurately would be higher.

$$F = \frac{MSR}{MSE} \quad (6)$$

Table 3. Analysis of variance (ANOVA)

| Source of Variation | Degree of Freedom | Sum of squares | Mean square | Statistic |
|---------------------|-------------------|----------------|-------------|-----------|
| Regression | $p-1$ | SSR | MSR | F |
| Residual | $n-p$ | SSE | MSE | |
| Total | $n-1$ | SST | | |

4. Experiment and Analysis

RSM is a statistics and mathematics technology that can be utilized to develop, improve, and optimize manufacture procedure. RSM design consists of three parts: experiment design, model construction, and parameter optimization. Experiment design is to obtain good quality distinction information by the few experiments. Model construction is to establish the functional relation between independent variable and response in experiment. The questions of quality design usually have some specific object to be optimized, which is achieved by parameter optimization. The first step is to determine the quality characteristics and control factors. The second step is to establish experiment method efficiently. The third step is to build approximate real system model by experiment result. The fourth step is to optimize the design model to get the optimal value. The last step is to simulate the optimized design to prove if it consists with expectancy [10][11].

5. Optimization Methods and Analysis

There are three control factors in this paper. The corner-point is designed by full factorial which total number of experiments is 2^3 . In the axial points α is $\sqrt[4]{2^3} = 1.682$. This design has done by four experiments in the central point experiment in order to increase the estimate of error. The experimental results using central composite design are shown in Table 4. T_c is peak-to-peak cogging torque. T_r is the ratio of average running torque to peak-to-peak running torque in steady state. T_c/T_r is the ratio of T_c to T_r . Using regression analysis on Table 4 can obtain second-order factorial regression model. The second-order factorial regression model is shown as follows:

$$y = \beta_0 + \sum_{i=1}^k \beta_i x_i + \sum_{i=1}^k \sum_{j=i+1}^k \beta_{ij} x_i x_j + \sum_{i=1}^k \beta_{ii} x_i^2 + \varepsilon \tag{7}$$

The fitted second-order function using regression analysis is

$$y = -0.15 + 3.32x_1 + 3.23x_2 - 2.86x_3 + 5.14x_1x_2 - 4.77x_1x_3 + 1.428x_1^2 + 1.19x_2^2 + 1.07x_3^2 \tag{8}$$

ANOVA is shown in Table 5 to discriminate the regression model obtained whether precise or not. The second-order function shown in Eq. (8) is optimized by adaptive genetic algorithm. The optimal code values obtained are $x_1 = -1.31$, $x_2 = -1.345$, and $x_3 = 0.583$, then these values translate to natural variables $x_1 = -12$, $x_2 = 5$, and $x_3 = 17$. The flux distribution of the optimized model is shown in Figure 4. The flux densities are considerably decreased in the vicinity of the magnet pole terminals, which also corresponds to reducing the magnetic saturation. Figure 5 and Figure 6 show the cogging

torque characteristics of the original design and the optimized design, respectively. Figure 7 and Figure 8 show the running torque characteristics of the original design and the optimized design, respectively. The performance comparison between the original design and the RSM design is shown in Table 6. The peak-to-peak value of the cogging torque of the original design is 0.205 N-m, but the optimized design is 0.139 N-m. From the results of FE analysis, the reduction of cogging torque by rotor pole design is more than 32.5%. The ratio of peak-to-peak torque ripple to average running torque is 0.289 before optimization, but it's 0.125 after optimization that amends torque ripple 56.75% definitely. It is found that the position and the shape of magnet poles, and the shape of notches are very effective for the reduction of cogging torque.

Table 4. Experimental results using central composite design

| T | θ_p | θ_r | T_c | T_r | T_c/T_r |
|--------|------------|------------|-------|-------|-----------|
| -1 | -1 | -1 | 0.471 | 4.286 | 0.110 |
| 1 | -1 | -1 | 0.728 | 3.333 | 0.218 |
| -1 | 1 | -1 | 0.418 | 0.632 | 0.661 |
| 1 | 1 | -1 | 0.811 | 0.020 | 40.55 |
| -1 | -1 | 1 | 0.115 | 4.615 | 0.025 |
| 1 | -1 | 1 | 0.911 | 4.000 | 0.228 |
| -1 | 1 | 1 | 0.219 | 0.805 | 0.273 |
| 1 | 1 | 1 | 0.87 | 0.462 | 1.883 |
| 1.682 | 0 | 0 | 1.125 | 0.511 | 2.202 |
| -1.682 | 0 | 0 | 0.351 | 4.211 | 0.083 |
| 0 | 1.682 | 0 | 0.4 | 0.440 | 0.909 |
| 0 | -1.682 | 0 | 0.371 | 6.000 | 0.062 |
| 0 | 0 | 1.682 | 0.621 | 3.810 | 0.163 |
| 0 | 0 | -1.682 | 0.115 | 0.930 | 0.124 |
| 0 | 0 | 0 | 0.472 | 5.714 | 0.083 |
| 0 | 0 | 0 | 0.475 | 5.714 | 0.083 |
| 0 | 0 | 0 | 0.479 | 5.714 | 0.084 |
| 0 | 0 | 0 | 0.481 | 5.714 | 0.084 |

Table 5. ANOVA obtains from experiment

| Source of Variation | Degree of Freedom | Sum of squares | Mean squares | Statistic |
|---------------------|-------------------|----------------|--------------|-----------|
| Regression | 9 | 1031.6 | 114.6 | 14.0732 |
| Residual | 8 | 495.53 | 61.94 | |
| Total | 17 | 1527.1 | | |

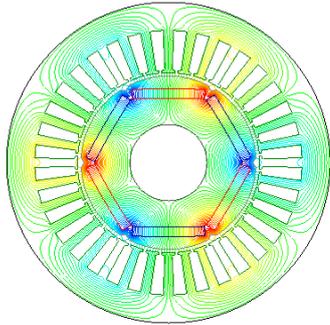


Figure4: Flux distribution of the optimized design

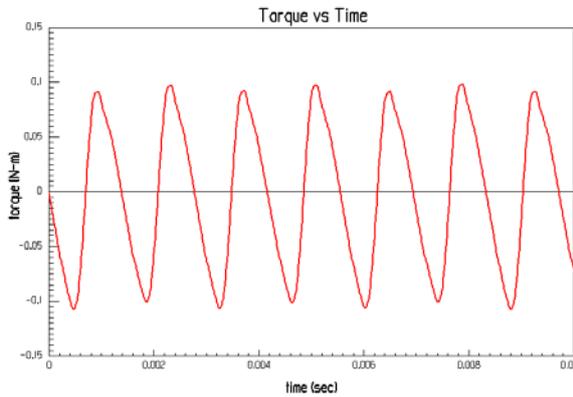


Figure 5: Cogging torque characteristics of original design

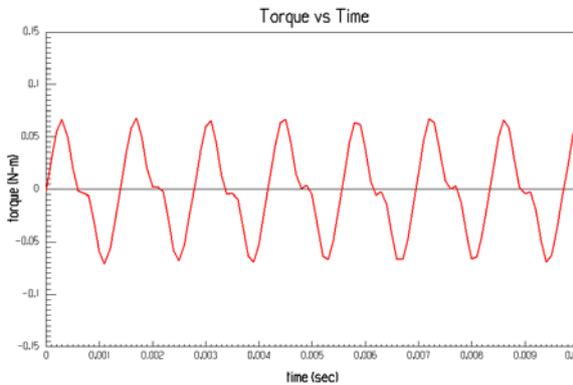


Figure 6: Cogging torque characteristics of optimized design

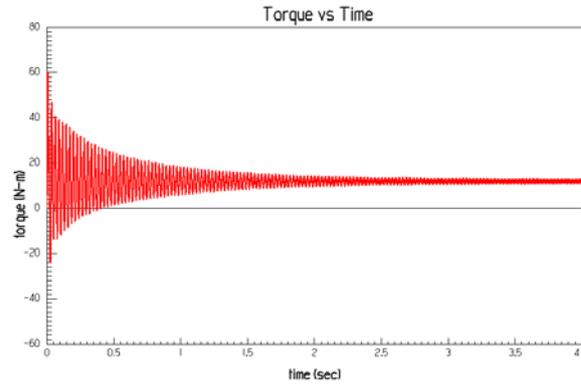


Figure 7: Running torque of original design

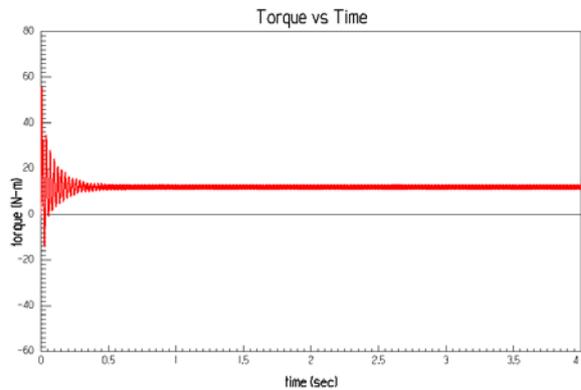


Figure 8: Running torque of optimized design

Table6. Performance comparison between original design and RSM design.

| | Original | RSM |
|-------------------------------|----------|---------|
| Cogging torque (peak-to-peak) | 0.205 | 0.139 |
| Average running torque | 12.8028 | 12.7864 |
| Torque ripple (peak-to-peak) | 3.7 | 1.6 |

6. Conclusions

In order to reduce the cogging torque of the IPMSM, this paper presents the optimal magnet pole design using the response surface method, and investigates the performance of the optimal designed, which confirms that the optimal design technique using RSM is well adapted to reduce the cogging torque. The results show that RSM can provide a realistic optimization capability in the design process for IPMSM.

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