PI Controller Applied in a Signal Security System Using Synchronous Chaos of Chua's Circuit

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Abstract

This paper aims to study how the chaotic phenomena are applied in the signal security system. This study uses the two identical Chua's circuits, one called master and the other called slave, so the slave added with PI controller can be synchronized in phase and amplitude with the master. To complete the signal encryption and decryption functions, the master is encrypted with a sine wave, and the slave can be decrypted with phase subtraction through PI controller. This article not only has the mathematical formula inference, but also provides circuit simulations to ensure the feasibility of the study.

Keywords: Chua's circuit, signal security system, synchronous, PI controller, chaos phenomenon

1. Introduction

Chaos phenomenon is an extremely complex dynamic nonlinear system, which behaves a long non-cyclical behavior and has a broad of Fourier spectrum. Chaos phenomenon has four distinct characteristics: decisiveness, sensitive to an initial value, singular Yukiko (Figure 1) and the track never to repeat. Decisiveness represents that chaos evolution seems a chaotic nonlinear reaction, but it follows certain equations to process. As sensitive to an initial value, it makes the trajectories in a chaos system, even if they are very close initially, they will be separated fast and exponentially [5,7], The chaos has the characteristics mentioned above, so chaos is applied to signal security to achieve the desired effect.

In a chaos synchronization system, comprising master (transmitting end) and slave (receiving end), the master provides the subsystem of a synchronization signal, and the slave is provided with the subsystem of a synchronization signal. To make chaos in both ends synchronized, controller design is very important. In recent years, many different control methods have been proposed one after another. PI controller is used in this paper due to its simple structure, stability control and robust features; so far it is still the most important industrial control tool.

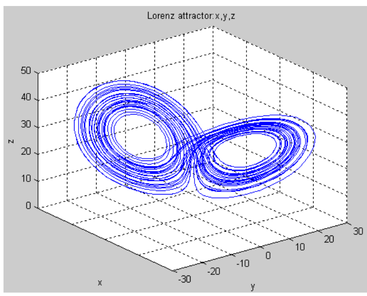

Figure 1: Lorenz chaos diagram by Matlab simulation

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2. Chua's Circuit Security System by Mathematics Simulation

2.1 Chua's Circuit Transformed Into a Three-dimensional Equation

Chua's chaos phenomenon by the circuit is not the same as the other chaos phenomenon by the equation.

In order to control this chaos system, Chua's circuit is transformed into three-dimensional equations. From Literatures [1,6], Chua's circuit diagram is shown in Figure 2. It is composed of simple linear devices including two capacitors, inductors, and a non-linear resistor (NR), where R is the resistance of the variable resistor. If the electronic parts implemented have errors, through fine tuning the variable resistor, the system can achieve stability.

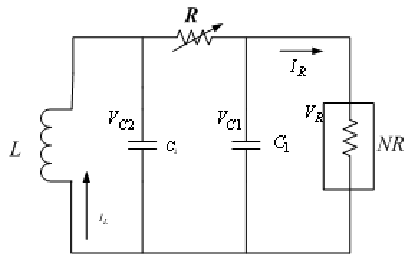

Figure 2: Chua's circuit architecture

The circuit through Kirchhoff's Law is transformed into three-dimensional equations, as expressed in Eqs (1) ~ (4):

$$C_1 \frac{dv_{C1}}{dt} = \frac{1}{R}(v_{C2} - v_{C1}) - f(v_{C1}) \qquad (1)$$

$$C_2 \frac{dv_{C2}}{dt} = \frac{1}{R}(v_{C1} - v_{C2}) + i_L \qquad (2)$$

$$L \frac{di_L}{dt} = -v_{C2} \qquad (3)$$

$$f(v_{c1}) = G_b v_{c1} + \frac{1}{2}(G_a - G_b)\{|v_{c1} + B_P| - |v_{c1} - B_p|\} \qquad (4)$$

Where Eq. (4) is a piecewise nonlinear characteristic equation, G_a and G_b are slopes, and the turning point is located between B_P and $-B_P$. This piecewise nonlinear V-I characteristic curve is shown in Figure 3 [6].

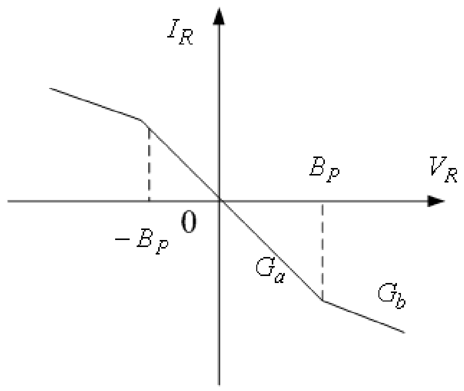

Figure 3: V-I characteristic of a nonlinear circuit

A nonlinear resistor can be presented in many methods. This paper uses a dual op amp circuit [8,9], as shown in Figure 4:

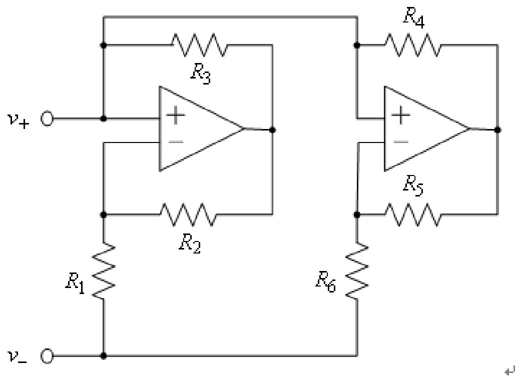

Figure 4: Dual OP amplifier circuit of nonlinear resistors

After algebraic conversion, three-dimensional equations are as follows:

$$\frac{dx}{d\tau} = \alpha(y - x - f(x)) \quad (5)$$

$$\frac{dy}{d\tau} = x - y + z \quad (6)$$

$$\frac{dz}{d\tau} = -\beta y \quad (7)$$

$$f(x) = bx + \frac{1}{2}(a-b)\{|x+1| - |x-1|\} \quad (8)$$

Where

$$x = \frac{v_{c1}}{B_P} 、 \quad y = \frac{v_{c2}}{B_P} 、 \quad z = \frac{i_L R}{B_P} \quad (9)$$

$$a = RG_a 、 b = RG_b 、 \tau = \frac{t}{|RC_2|} 、 \alpha = \frac{C_2}{C_1} 、 \beta = \frac{R^2 C_2}{L} \quad (10)$$

Where x, y, z are real variables, α、β、a、b are constants. After the known values are substituted into Eq. (10) , it can obtain α =14、β =21、a=*-1.32* and *b=-0.84.*

2.2 The Flow Chart of Chaos Signal Security System is Shown in Figure 5

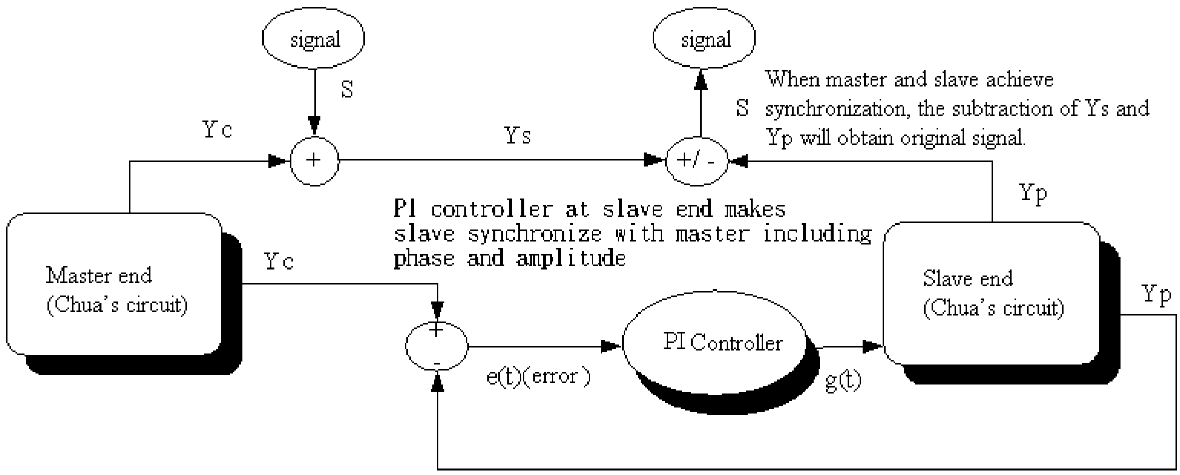

Figure 5: chaos signal security system flowchart

Where Y_c is the output of Y-axis at master; Y_p is the output of Y-axis at slave; Y_s is the result of Y_c added to the signal (S); e(t) is the error of Y_s -Y_p. For PI controller architecture diagram, please refer to the next section.

2.3 PI Controller

In the control system, PI controller is the simplest controller, and PI control system is shown in Figure 6.

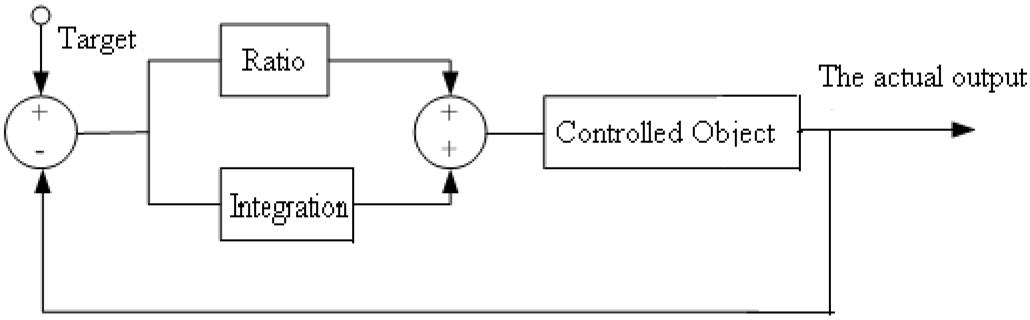

Figure 6: PI controller architecture diagram

The equation of PI controller is expressed in Eq. (11)

$$g(t) = kp\left[error(t) + \frac{1}{T_i}\int_0^t error(t)\Delta t \right] \quad (11)$$

Where g (t) is the control input of the slave side of the chaos system; k_p is the proportional coefficient; T_i is the integral time constant; error (t) is the error value of $y_c - y_p$.

$$k_i = k_p \times \frac{1}{T_i} \quad (12)$$

Where ki is the integral controller gain

After Eq. (12) is substituted into Eq. (11), we obtain

$$g(t) = kp(error(t)) + ki\int_0^t error(t)\Delta t \quad (13)$$

2.4 Simulink Simulation Signal Security System

First, the master mathematical formula are expressed as Eqs (14) ~(16); the slave mathematical formula are expressed as Eqs (17) - (19); PI controller mathematical formula is expressed as Eq. (13).

$$\frac{dx_c}{d\tau} = \alpha(y_c - x_c - f(x_c)) \quad (14)$$

$$\frac{dy_c}{d\tau} = x_c - y_c + z_c \quad (15)$$

$$\frac{dz_c}{d\tau} = -\beta y_c \quad (16)$$

$$\frac{dx_p}{d\tau} = \alpha(y_p - x_p - f(x_p)) \quad (17)$$

$$\frac{dy_p}{d\tau} = x_p - y_p + z_p + g(t) \quad (18)$$

$$\frac{dz_p}{d\tau} = -\beta y_p \quad (19)$$

The simulink block diagram implemented by mathematical formulas of master, slave and PI controller are shown in Figure 7.

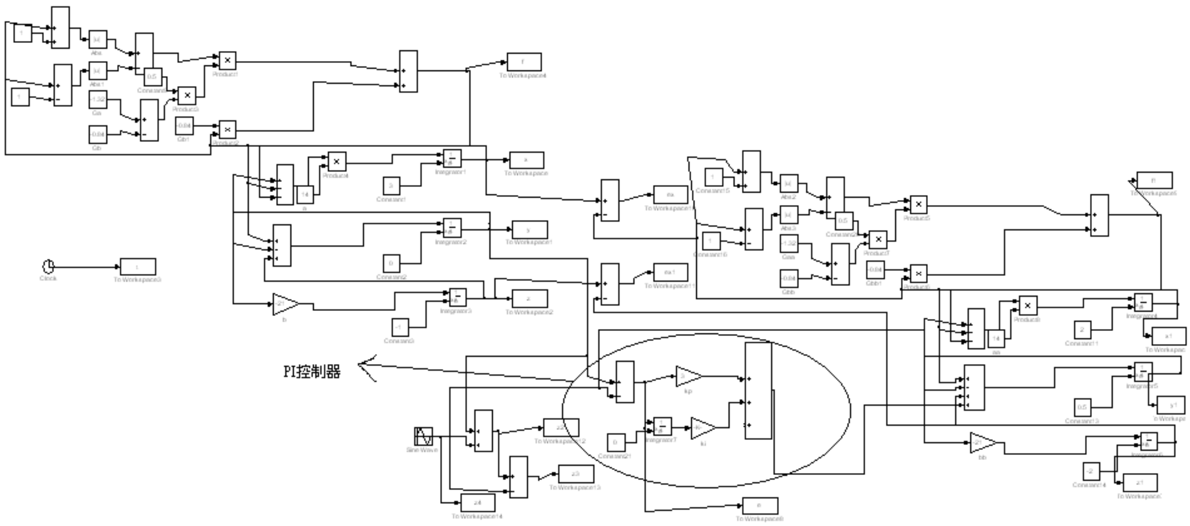

Figure 7: the mathematics simulation diagram of chaos security system

A group of best parameters obtained by Matlab simulation, k_p =3 and k_i =1.001, make the PI controller achieve the best control.

Chaos waveforms of Chua's circuit are shown in Figure 8, which confirms that the waveforms by Chua's circuit can be converted into three-dimensional equations to simulate the Chua's circuit Chaos phenomenon.

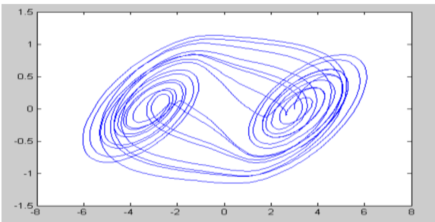

Figure 8: Chua's chaos waveforms

Figures 9~ 11 are synchronization errors between x_c and x_p、 y_c and y_p、 z_c and z_p, respectively.

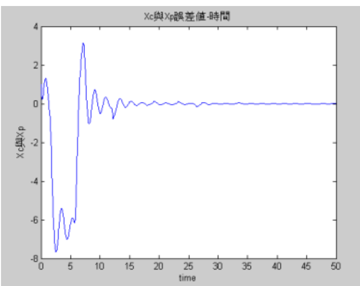

Figure 9: x_c and x_p synchronization error

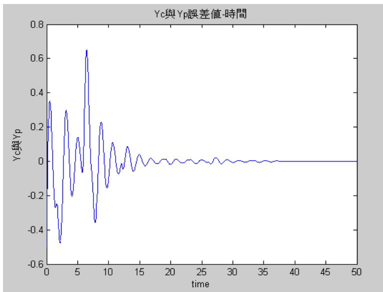

Figure 10: y_c and y_p synchronization error

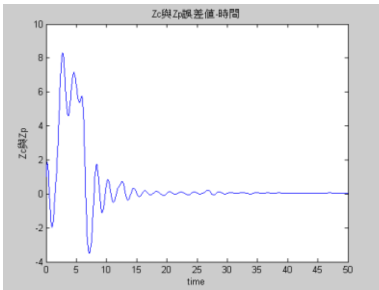

Figure 11: z_c and z_p synchronization error

From the above figures，it shows that in about 10 seconds the synchronization error values are stable and close to 0. That means that in about 10 seconds the output waveforms can be synchronized. Figures 12-17 show the comparison between before synchronization and after synchronization waveforms.

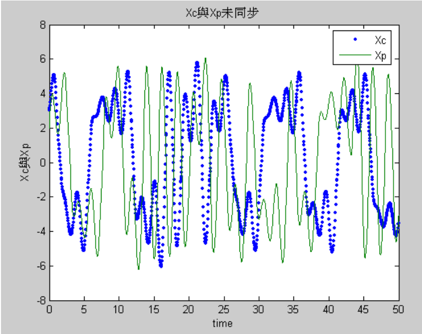

Figure 12: x_c and x_p non synchronization error

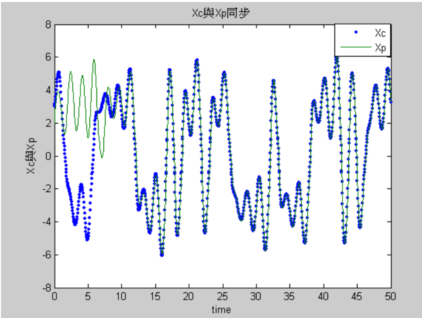

Figure 13: x_c and x_p synchronization error

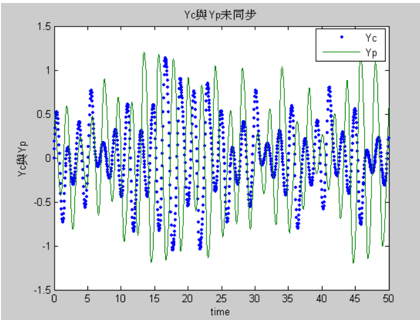

Figure 14: y_c and y_p non synchronization error

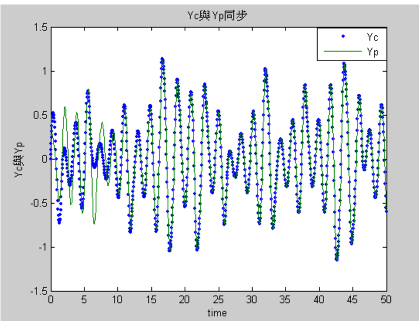

Figure 15: y_c and y_p synchronization error

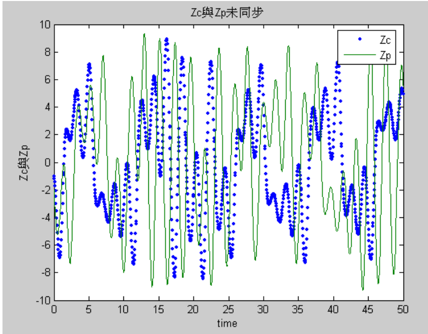

Figure 16: z_c and z_p non synchronization error

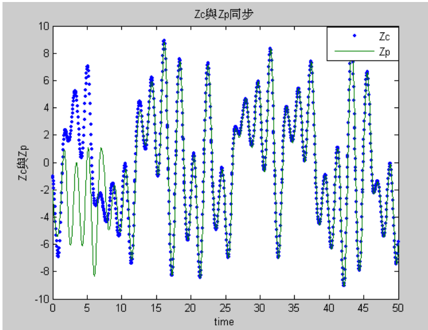

Figure 17: z_c and z_p synchronization error

Adding a sine wave and master together can mess a sine wave signal without rules (signal encryption), and then subtracting between synchronized master and slave can finally get back to the sine wave (signal decryption), as simulated in Figures 18 and 19.

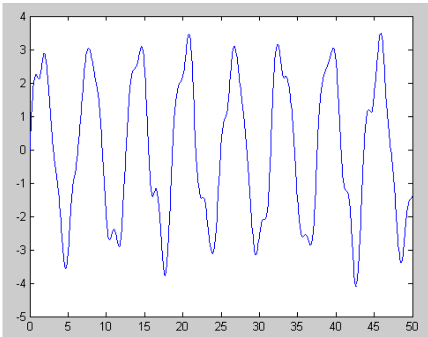

Figure 18: waveform after sine wave encrypted

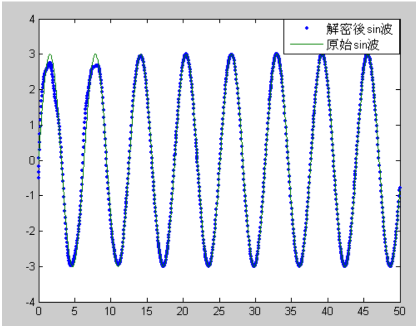

Figure 19: comparison chart of decrypted sine wave with the original sine

From the above figures, it is apparent that the two waveforms overlap in approximately 10 seconds, representing the decrypted sin wave is returned to the original sin wave.

2.5 Security System of Chaos Circuit Using fPSpice Simulation

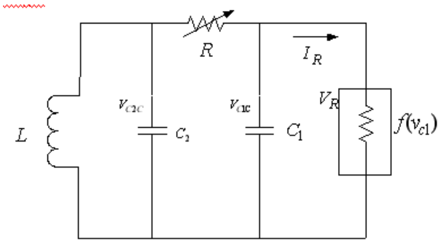

Figure 20: master subsystem

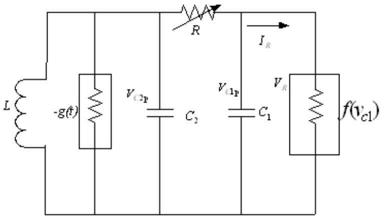

Figure 21: slave subsystem

Chua's master and slave subsystems as shown in Figures 20~21 convert PI controller mathematics, and the operation of encryption and decryption into circuits is based on OP amplifiers, as shown in Figures 22 ~ 24:

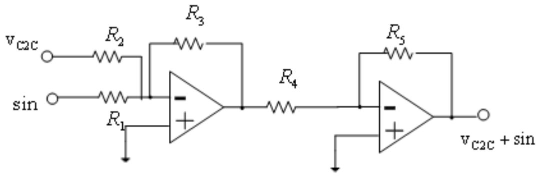

Figure 22: Encryption circuit

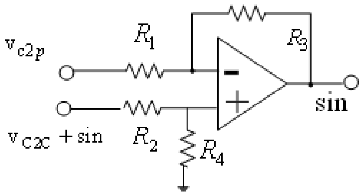

Figure 23: Decryption circuit

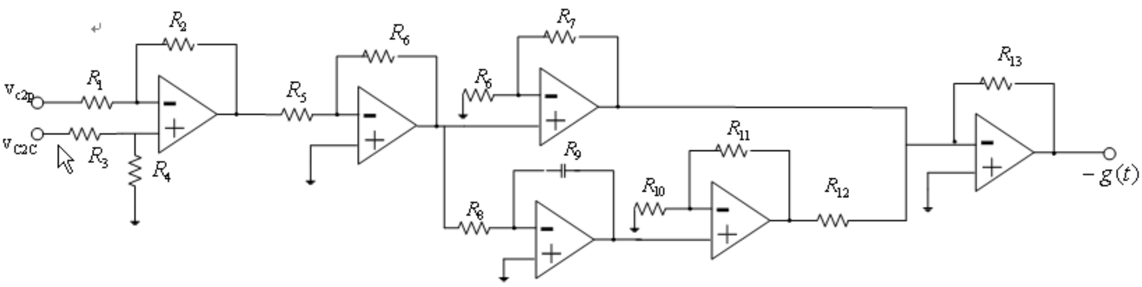

Figure 24: PI controller circuit diagram

PSpice circuit simulation software is used to construct the circuit diagram as shown in Figure 25:

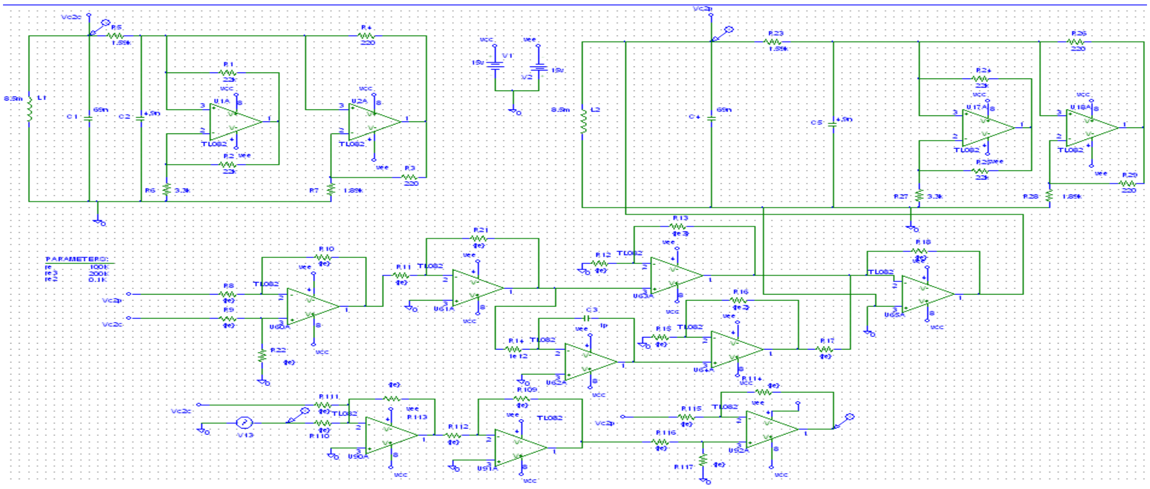

Figure 25: PSpice modeling circuit

Chua's chaos waveform of Chua's circuit by PSpice simulation is shown in Figure 26.

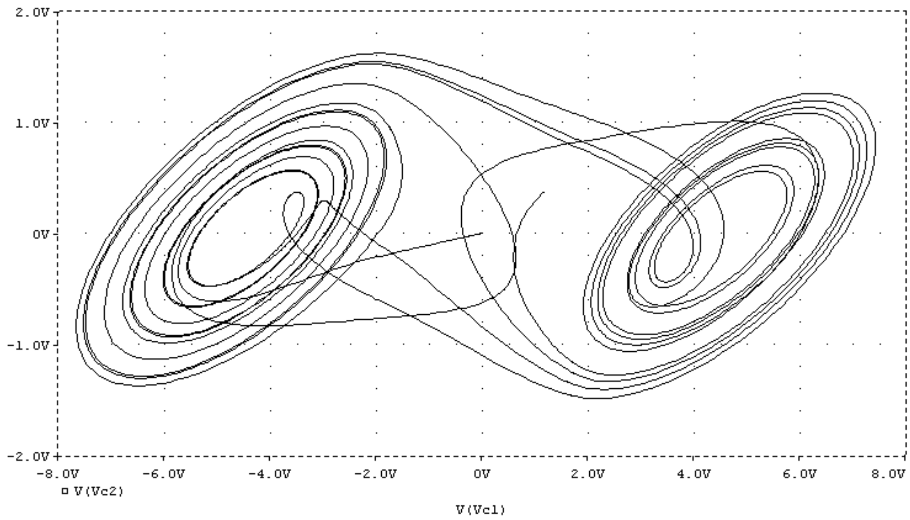

Figure 26: - Output waveforms of V_{C1C} - V_{C2C}

The comparisons between simulations with and without PI controller, which are equivalent to synchronization and non-synchronization, respectively, are shown in Figures 27~ 30

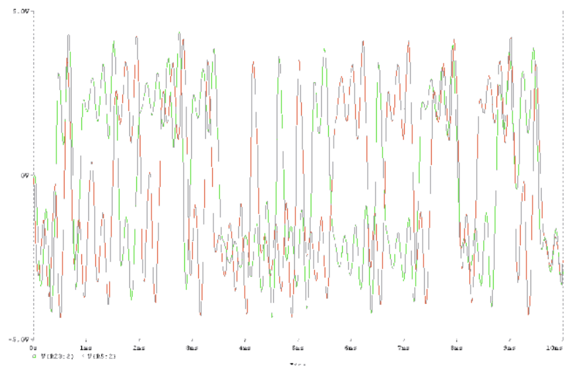

Figure 27: waveform of not synchronized V_{C1C} and V_{C1P}

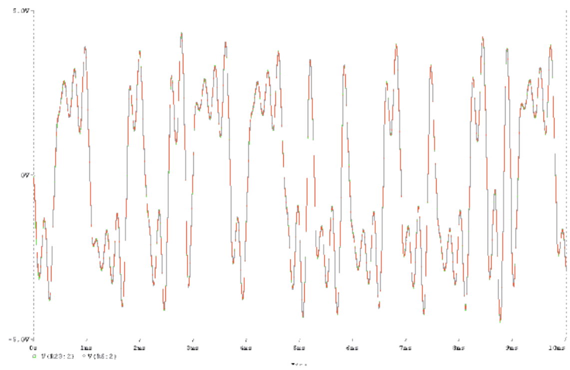

Figure 28: waveform of synchronized V_{C1C} and V_{C1P}

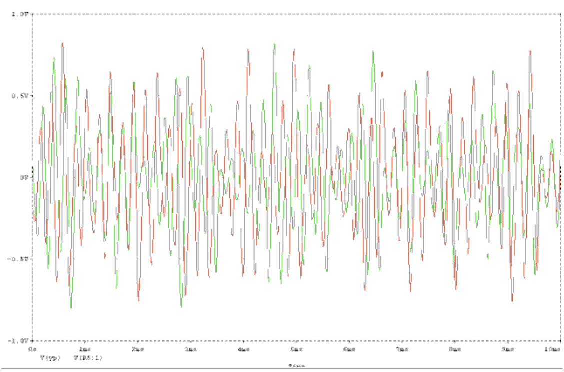

Figure 29: waveform of not Synchronized V_{C2C} and V_{C2P}

Figure 30: waveform of synchronized V_{C2C} and V_{C2P}

After master added with a sine wave messes a sine wave (encryption), subtracting synchronized master from slave can obtain the original sine wave (decryption), as shown in Figures 31 - 33:

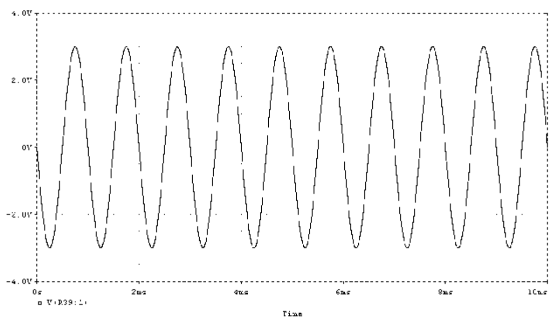

Figure 31: original sine wave

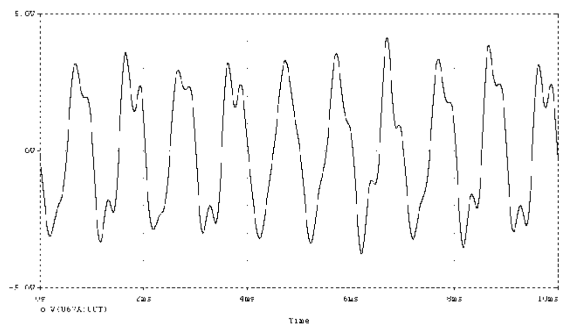

Figure 32: encrypted sine wave

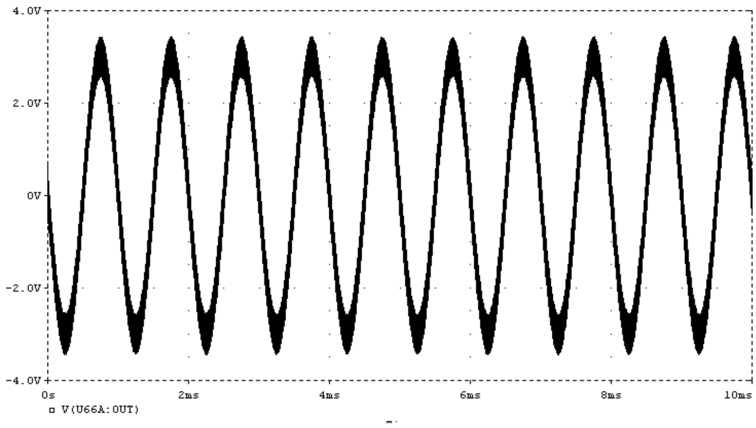

Figure 33: decrypted sine wave

3. Results and Discussion

In this paper, after simulations by the mathematical formula and circuits separately, it is observed that when chaos is added to the original signal, there is no regularity in the original signal. It cannot be solved without finding the appropriate control methods. Through a simple and easy PI controller, it makes the idea of synchronizing two Chua's chaos implemented, so it can be found Chaos is very suitable for a security system. This article has demonstrated that chaos used in communication security is even more advanced further in the future.

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