

Denoising Algorithms Based on EMD and Wavelets

^{1,2,*} Shiru Zhang, ¹Xiaotong Chen, ¹Qingfu Sun and ¹Caiying Zhang

Abstract

Pulse signals are one of the most important physiological signals, containing a large number of physiological and pathological information of human body. Various disease information of the human body is often contained in their pulse signals. The change of the pulse signal characteristics is often the earliest embodiment of human disease. Research of human pulse signals is very helpful to disease diagnosis. Human pulse signals are often corrupted with noise, and it is difficult to extract the clean pulse signals. Therefore, denoising is a very important and difficult job before pulse signal analysis. This paper starts at pulse signal analysis and focuses on denoising algorithms.

EMD (empirical mode decomposition) and a wavelet transform denoising method are described first, and then the advantages and disadvantages of them are analyzed; a novel denoising algorithm combining EMD and a wavelet transform is proposed. Finally the proposed algorithm is compared with wavelet denoising and the EMD denoising method qualitatively and quantitatively.

Keywords: Pulse signal, Wavelet transform, Empirical mode decomposition (EMD), de-noising

1. Introduction

In the process of human pulse signal acquisition, the examinee's breathing, body displacement, the noise of the instrument itself, and the other factors will directly affect the quality of the acquired pulse signals. If no preprocessing required to the corrupted signals is done to minimize the effects of the noises or interferences on the pulse signal, the subsequent analysis will not be accurate.

Concerning the pulse signal denoising method, the EMD algorithm and the wavelet threshold algorithm are commonly used in recent years and have achieved good results. But in the wavelet algorithm there are many factors, and changing one of the factors will lead to different denoising effects.

In EMD method its multi-resolution and the adaptability of decompositions are very useful in nonlinear and non-stationary signal processing. However, in EMD denoising process all the high frequency components are removed, so this will lead to the loss of useful information. Therefore, this method is too rough although it is simple. This paper combines the good time-frequency localization characteristics of the wavelet transform with the simplicity of the EMD method, and proposes a novel algorithm applicable to denoising human pulse signals.

2. EMD Method

2.1 The EMD Principle

In everyday life, most signals are complex signals with multiple components, and it is difficult to analyze their characteristics. As a result, the multi-component signal is decomposed into multiple signals with single component, and after processing it will be reconstructed in order to get the original signal type. The essence of the EMD method is to obtain intrinsic mode functions (IMF) with different instantaneous frequencies through the characteristic time scales of the signal, and then the signal is decomposed.

The procedure of the EMD decomposition is: 1) to find out the points of local maximum and minimum value of the signal; 2) to apply curve interpolation to obtain the extremum points and the upper envelopes, lower envelope and the mean envelope; 3) to select signals conforming to the intrinsic mode functions of the signal by sifting process; 4) to decompose the signal into the sum of several intrinsic mode functions and a residual. There is no direct relationship between IMFs. The EMD is a kind of signal sifting method. The frequencies of the decomposed IMF are from high to low. The upper and lower envelopes generated by curve interpolations in the decomposition process from the local maxima and minima are shown in Figure 1. In Figure 1 the black thin line represents the original signal; the short dotted line on the top is the upper envelope; the short dotted line in the bottom is the lower envelope; and the thick dashed line in the middle is the mean envelope of the signal.

*Corresponding Author: Shiru Zhang
(E-mail: zsr0504@qq.com)

¹College of Communications and Information Engineering, Xi'an University of Science and Technology, No.58 Yanta Road, Yanta District, Xi'an, Shaanxi, 710054, P. R. China

²Department of Electronic Engineering, National Chin-Yi University of Technology, No.57, Sec.2, Chung-Shan Rd., Taiping District, Taichung, City 41170, Taiwan

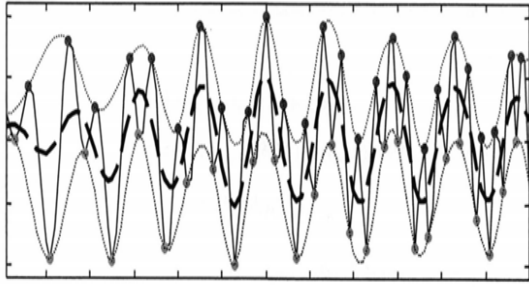


Figure 1: Upper envelope, lower envelope and the mean envelope of a signal

In conclusion, the sifting procedure of the decomposed IMF is shown in Figure 2.

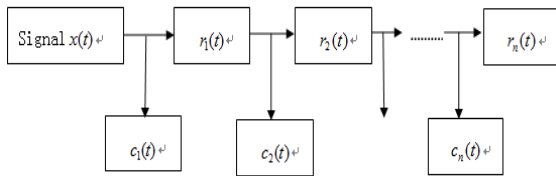


Figure 2: The EMD of the screening process diagram

In each sifting process an IMF will be selected, and in the next sifting process the rest signal is sifted. When all the sifting processes are finished, the original signal can be expressed as shown in (1) where $r_n(t)$ is the last remaining trend information, and $c_i(t)$ is the i th IMF component.

$$x(t) = \sum_{i=1}^n c_i(t) + r_n(t) \quad (1)$$

2.2 The Application of EMD in Signal De-noising

In EMD method, a data sequence can be decomposed into different frequency channels whose frequency component is listed from high to low. Due to the high frequency characteristics of noise, it is sure that EMD will have a wide application in signal denoising. In principle, EMD method not only has the advantage of multi-resolution similar to that of wavelet transform, but also overcomes the disadvantage of choosing wavelet base. Also, it does not need to choose the number of decomposition layer. EMD can decompose a signal adaptively according to the nature of the signal itself.

Wu and Huang [1,2] found the statistical properties of white noise in EMD algorithm through extensive experimental studies: After EMD decomposition each IMF component of a white noise signal conforms to a normal distribution. Every Fourier spectrum of IMF is consistent, covering the same area under semi log scale coordinates. In addition, Wu and others got the conclusions that the

product of the average energy density of an IMF and its corresponding cycle keeps a constant, and the average cycle is about twice the previous IMF component of the cycle. That is:

$$E_i \bar{T}_i = const \quad (2)$$

$$\bar{T}_i = 2\bar{T}_{i-1} \quad (3)$$

$$\text{where } E_i = \frac{1}{N} \sum_{j=1}^N [IMF_i(j)] \quad (4)$$

In the above equations E_i represents the energy density of the i th IMF, N is the data length, \bar{T}_i is the average cycle of the i th IMF, which is defined as the ratio of the number of the data sampling and the number of maxima in the i th IMF components. That is $\bar{T}_i = N / N_{max}$, where N_{max} is the number of maxima in the i th IMF. All of the above provide a theoretical basis for the EMD denoising.

2.2.1 EMD Scale De-noising

EMD sifts signals layer by layer by different time scales. Each IMF corresponds to a specific time feature scale, and it can be used to the signal filtering. The former of the IMF is decomposed, and the higher frequency it will have. The first decomposed frequency is the highest frequency of the original signal. The later frequencies decomposed by EMD will be intermediate frequency and lower frequency components, and the last one is a monophonic trend component. For an original signal containing noise, the high frequency resulting from the decomposition of the IMF component is often the noise of the signal, while the low frequency component of IMF is generally the average or the trend of the original signal. For the purpose of convenient analysis, we can combine arbitrary intrinsic mode functions to highlight some characteristics in a certain frequency range of the analyzed signal. Flandrin proposed a method of constructing a filter bank based on the filtering characteristics of EMD decomposition [3,4,5]. For instance, through EMD method a signal is decomposed into n intrinsic mode components. As a matter of convenience the last trend term is assumed to be the n th IMF. So a low-pass filter can be obtained by removing the first or several former IMF components, and combining the remaining IMF components, as shown in Eq. (4).

$$x_{ik}(t) = \sum_{j=k}^n c_j(t) \quad (k < n) \quad (4)$$

A high-pass filter is obtained by removing the trend term or the last few low-frequency IMF and combining the rest of IMF, as shown in Eq. (5).

$$x_{hh}(t) = \sum_{j=1}^h c_j(t) \quad (h < n) \quad (5)$$

A band-pass filter is obtained by removing the first or the former few IMFs and the last one or several IMFs, and combining the rest of IMFs, as shown in Eq. (6).

$$x_{bhk}(t) = \sum_{j=h}^k c_j(t) \quad (1 < h < k < n) \quad (6)$$

A band-stop filter can be obtained by removing several middle terms of intermediate frequency of IMF, and combining the remaining higher frequency and the lower frequency components, as shown in Eq. (7).

$$x_{bshk}(t) = \sum_{j=1}^h c_j(t) + \sum_{j=k}^n c_j(t) \quad (1 < h < k < n) \quad (7)$$

It can be seen that the EMD denoising method is very simple. The filter bank is different from the traditional fixed cutoff frequency filters. The passband cutoff frequency will change with the input signal, so in reality it is an adaptive filter banks. The obvious advantage is that, due to the IMF decomposition the nonlinear and the non-stationary characteristics of the signal are preserved to the largest extent after filtering. Also there is no limit to the data type. Because the selection of the base functions is in the signal itself, it does not need to define the parameters of the filter. Therefore, the influence of subjective factors is reduced, and better results will be obtained in most cases.

However this denoising approach is relatively rough. In one hand there are not only noises but also signals in some IMF components. Simply removing a few components may lead to the loss of useful signals. On the other hand the number of IMF component is not known in advance, and the position of the noise in IMF component is not known. Therefore, whichever component will be removed is very difficult to determine.

2.2.2 EMD Thresholding

After EMD decomposition, noises and the useful signals will have different spectrum characteristics, and random noises are mainly in the low-order of IMF component. If the low-order of the IMF component is only removed, and the higher order and the other components are preserved, most of the noises will be removed. However, this denoising method will filter out the useful signal at the same time. Therefore, the signal will be distorted.

The energy of a random noise is uniformly distributed, and the energy of a useful signal is relatively concentrated. Therefore, the absolute value of the IMF coefficients of the noise is small, and the IMF coefficients of the useful signal are relatively large. If we take the absolute value of the IMF coefficient as a local measurement and designate a reasonable threshold standard, we can practice thresholding denoising method like this: If the absolute value of the coefficient is smaller than the threshold, we think it belongs to a noise and let this coefficient be zero. If the absolute value of the coefficient is bigger than the threshold, we think it belongs to a useful signal.

For a given signal $x(t)$, we can get n IMF components through EMD decomposition. For each layer of the IMF component an appropriate threshold is specified. Assume the IMF components are $c_i(t)$, and after threshold processing it will be $c_i'(t)$. This signal can be reconstructed by Eq.(8).

$$x'(t) = \sum_{i=1}^n c_i'(t) + r_n(t) \quad (8)$$

Where the threshold processed IMF components are as shown in Eq. (9).

$$c_i'(t) = \begin{cases} \text{sgn}\{c_i(t)\}(|c_i(t)| - T), & |c_i(t)| \geq T \\ 0, & |c_i(t)| < T \end{cases} \quad (9)$$

Where T is the threshold, $\text{sgn}(\cdot)$ is a sign function.

3. Proposed De-noising Algorithm

3.1 Denoising by Wavelets

With regard to a wavelet transform, its multi-resolution and time-frequency local properties, together with the fast algorithm, all make it a concern spot in denoising area.

3.1.1 Wavelet Thresholding

A wavelet thresholding method was first proposed by Weaver et al [6], then Donoho and Johnstone [7], Professors at Stanford University made this approach systematically explained. The main theoretical basis is that an orthogonal wavelet transform has a strong decorrelation feature. After a wavelet transform, the signal energy focuses on the large wavelet coefficients, and the noise energy distributes throughout the wavelet region. When the wavelet coefficient is less than the critical threshold, it is mainly caused by the noise and should be discarded. When it is greater than this critical threshold, it is mainly caused by the signal. And then these coefficients are processed to form a new group of coefficients. Finally the denoised signal will be obtained by wavelet reconstruction.

Wavelet thresholding process for one-dimensional signal can be divided into the following three steps:

- 1). Wavelet decomposition: It selects a wavelet and the decomposition level N, and practice the N level decomposition to the signal.
- 2). Threshold processing: In order to keep the overall signal shape and preserve the low frequency coefficients, for the high frequency coefficients from the first to the Nth layer, quantization processing is done by thresholding method for each layer.
- 3). Signal reconstruction: For the low-frequency coefficients of the Nth layer and the high-frequency coefficients from the first to the Nth layer of the quantized process, inverse wavelet transform is done to get the denoised signal.

3.1.2 Selection of Threshold Function

Threshold function represents the different treatment strategies and different estimation methods for the wavelet coefficients which are less or greater than the threshold. There are two types of threshold functions: hard threshold function, soft threshold function. Suppose w is an original wavelet coefficient, $\eta(w)$ is the wavelet coefficient after thresholding, and T is the threshold, then

- 1). hard threshold function

$$\eta(w) = \begin{cases} 0, & |w| < T \\ w, & |w| \geq T \end{cases} \quad (10)$$

- 2). soft threshold function

$$\eta(w) = \begin{cases} \text{sgn}(w)(|w| - T), & |w| \geq T \\ 0, & |w| < T \end{cases} \quad (11)$$

Where $\text{sgn}(w)$ is the sign function, that is,

$$\text{sgn}(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases} \quad (12)$$

In Fig.3 and Fig.4, the abscissa represents the original signal wavelet coefficients, and the ordinate represents the wavelet coefficients after thresholding. In the hard threshold, if the absolute value of the wavelet coefficient is less than the threshold, the processed wavelet coefficient is forced to be zero. The disadvantage is that it will bring some discontinuous points, but it can better preserve the jump signal. Based on the hard threshold method, soft threshold method shrinks the upper boundary discontinuities to zero. Therefore, you can effectively avoid interruption, make the reconstructed signal relatively smoother. But soft threshold method will cause edge blurring distortion.

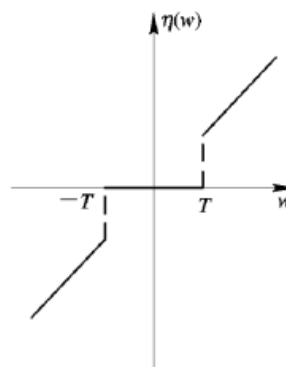


Figure 3: Hard threshold function

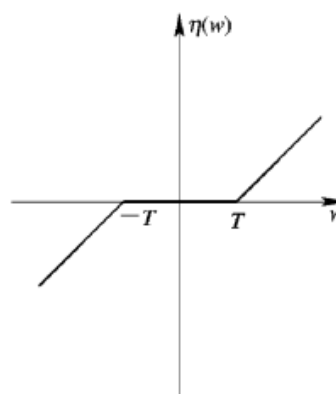


Figure 4: Soft threshold function

A wavelet thresholding method is simple and has small amount of calculation, so it gets extensive applications.

3.2 Improved Algorithm

This paper mainly focuses on the wavelet threshold denoising method, and proposes an improved algorithm decompose human pulse signal by EMD method; determine the dominant intrinsic mode function (IMF) component of noise; denoise these IMF components with an improved wavelet threshold method. The improved algorithm has advantages of continuity, high order derivative and good flexibility. It overcomes the defects of wavelet threshold denoising, avoid the difficulty of wavelet bases choice, and preserves the useful information in the original signal under the insurance of good denoising effect.

3.2.1 Determine the IMF components of denoising

Generally EMD wavelet threshold denoising method processes all the decomposed IMF components. This will lead to the loss of the useful signal. But this paper will process parts of the IMF components in order to avoid more distortion.

Boudraa A O [5] proposed a concept of continuous mean square error. The essence of it is the energy of every IMF component. The definition of continuous mean square error is as expressed by Equation (12):

$$CMSE(\tilde{x}_k, \tilde{x}_{k+1}) = \frac{1}{N} \sum_{i=1}^N [\tilde{x}_k(t_i) - \tilde{x}_{k+1}(t_i)]^2$$

$$= \frac{1}{N} \sum_{i=1}^N [IMF_k(t_i)]^2, \quad (k=1, \dots, n-1) \quad (12)$$

Where, N is the length of the signal, and $IMF_k(t_i)$ is the kth-order IMF decomposed by noise. The demarcation point k can be computed by Eq.(13).

$$k = \arg \min_{1 \leq k \leq n} [CMSE(\tilde{x}_k, \tilde{x}_{k+1})] + 1 \quad (13)$$

The former k IMF components is dominated by noise, and should be de-noised. Then all the IMF after the kth components need to be reconstructed:

$$x'(t) = \sum_{i=k+1}^N IMF_i(t) \quad (14)$$

3.2.2 Modified Wavelet Threshold Function

Although the soft, hard threshold method finds wide applications in wavelet denoising, there are some defects in the two algorithms. In hard threshold processing there are discontinuous points, and pseudo Gibbs phenomena will occur in the signal reconstruction. In soft threshold processing, although there are no continuity problem, a constant deviation will lead to an inevitable error for the reconstructed signal [8].

According to some theoretical analysis, we put forward the following improvement.

$$IMF'_j(t) = \begin{cases} uIMF_j(t) + (1-u)\text{sgn}(IMF_j(t))(|IMF_j(t)| - thr_j)^{\frac{1}{a}} & |IMF_j| \geq thr_j \\ 0 & |IMF_j| < 0 \end{cases} \quad (15)$$

In (15), $u = 1 - e^{-m(|IMF_j| - thr_j)^2}$, $m \geq 0$, $0 \leq u \leq 1$,

$$\text{sgn}(x) = \begin{cases} 1 & IMF_j > 0 \\ 0 & IMF_j = 0 \\ -1 & IMF_j < 0 \end{cases}$$

j represents the jth-order IMF, $a > 0$.

The modified threshold function is continuous. When $m \rightarrow 0$, $a=1$, it is a soft threshold function; when $m \rightarrow \infty$ it is a hard threshold function. As long as the parameter M and a are adjusted properly, we can get better denoising result.

Human pulse signal is decomposed into several IMF components by EMD. Find the IMF components in which noise is dominant and do threshold processing. First, calculate the thresholds of the IMF components which need to be denoised; and then process these IMF components through the modified threshold function. In this paper we use a modified algorithm (16) to estimate the threshold according to [8].

$$thr_j = \sigma_j \sqrt{2 \lg(N) / \lg(e + j - 1)} \quad (16)$$

Where, e is the natural logarithm. When $j=1$, the result is consistent with the original fixed calculation formula.

3.3 Algorithm Details

The block diagram of wavelet threshold denoising algorithm is shown in Fig.5. This method is summarized in the following steps:

- 1). Perform an EMD decomposition to the original noisy signal x (t), and N IMFs ($IMF_1 \sim IMF_N$) are obtained.

$$x(t) = \sum_{i=1}^n IMF_i(t) + r_n(t) \quad (17)$$

- 2). Since the first order IMF is almost all the noise, remove it directly;
- 3). Calculate continuous mean square error of each IMF component by Equation (12), and determine the cutoff point of k;
- 4). Find the IMF components ($IMF_1 \sim IMF_k$) in which noise dominants, and calculate the noise level in these components;

$$\sigma_j = \frac{\text{median}(|IMF_j(t)|)}{0.6745} \quad (18)$$

Where median () represents the value in the middle position if all the values in the sequence are listed from small to large or from large to small. If the sequence length is odd, then take the middle element; if the sequence length is even, then take the arithmetic average value of the two elements in the middle position.

- 5). Estimate the thresholds of the IMF components in which noise is dominant by using Equation (16), where N is the length of the signal, j means the jth IMF component ($j=1, 2, \dots, l$)
- 6). Do the wavelet threshold denoising to $IMF_1 \sim IMF_l$ by Equation (15).
- 7). Reconstruct the pulse signal $x'(t)$ by the denoised $IMF'_1 \sim IMF'_l$, the original signal component $IMF_{l+1} \sim IMF_n$ and the residual signal $r_n(t)$.

$$x'(t) = \sum_{j=1}^l IMF'_j(t) + \sum_{j=l+1}^n IMF_j + r_n(t) \quad (19)$$

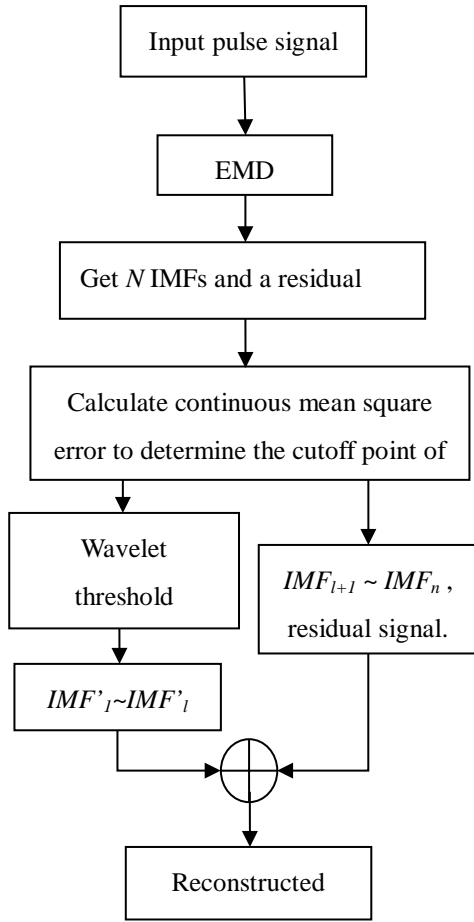


Figure 5: The block diagram of wavelet threshold denoising based on EMD

3.4 Effective Denoising Evaluation

Generally signal-to-noise ratio (SNR) and mean square error (MSE) are used to evaluate a denoising method. However, pulse signal contains rich pathology information, so we need add another index—cross-correlation coefficient to evaluate the whole performance.

1). signal-to-noise ratio (SNR)

$$SNR = 10 \log_{10} \left[\frac{\sum_{n=0}^{N-1} s^2(n)}{\left[s(n) - \hat{s}(n) \right]^2} \right] \quad (20)$$

Where $s(n)$ is the original signal, $\hat{s}(n)$ is the denoised signal. The larger of SNR is, the better the denoising effect is, and the less noise in the signal.

2).Mean square error (MSE)

$$MSE = \frac{1}{N} \sum_{n=0}^{N-1} \left[s(n) - \hat{s}(n) \right]^2 \quad (21)$$

Where n is the number of sampling points, $s(n)$ is the original signal, and $\hat{s}(n)$ is the de-noised signal. When MSE is smaller, the better denoising effect.

4. Simulation Results

4.1 Pulse Signal Acquisition

The tested human pulse signals in this paper are acquired by the pulse instrument designed by Wang Yamin, a teacher of Tianjin University in China.

According to TCM (traditional Chinese medicine) theory, the human wrist cunkou is divided into three parts: cun, guan, and chi; and different part of the pulse information represents the physiological and pathological information corresponding to the different position of the human body. The pulse pressure sensor is placed in some fixed position of the tester to collect the pulse signal. To test the three parts: cun, guan, and chi, on the left and right wrist, a total of six points need to be measured. The cunkou radial artery pulse on the left wrist is the strongest one, so we focus on it and process it in this paper.

4.2 De-noising Results of Human Pulse Signals

All the simulations in this paper are completed in the matlab7.0 platform. The sampling frequency is 1500Hz, and the number of sampling points is 7232.

Figure 6 shows a segment of the noisy pulse signal measured on the left Cunkou. This pulse signal is denoised by the proposed method. As can be seen from Figure 6, the edge of the original signal is not clear, and this means that there exists various kinds of noises.

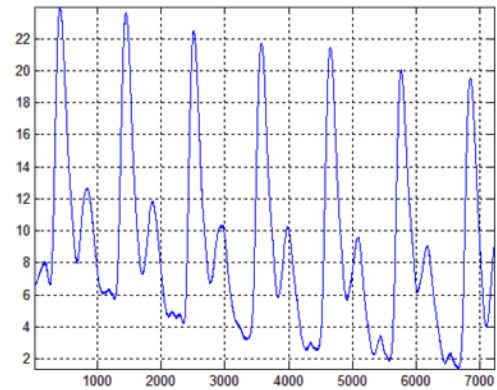


Figure 6: A segment of the noisy pulse signal measured on the left Cunkou

1).Wavelet thresholding

Get 8-scale decomposition by using sym8 wavelet, and then soft threshold to the noisy pulse signal by the fixed threshold rule. As we can see from Figure 7, after denoising by a soft threshold method, the pulse waveform becomes smoother. The primary noise is suppressed, and the peak points, which represent the signal's features, are well preserved. But at the discontinuous points, Gibbs phenomenon occurs in the edge of the signal, and there exists some waveform distortion.

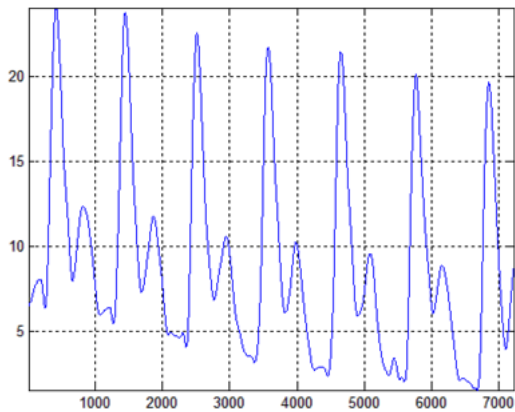


Figure 7: Pulse signal after wavelet de-noising

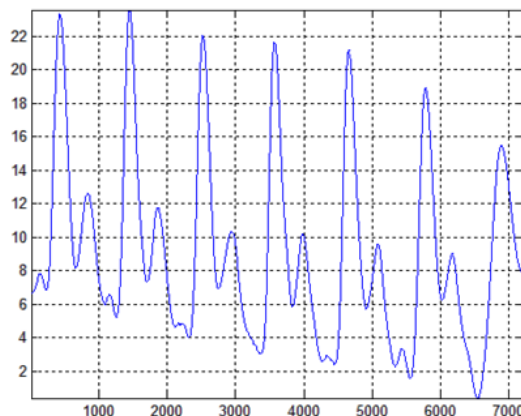


Figure 9: EMD de-noising

2).EMD de-noising

Performing EMD decomposition to the noisy signal in Figure 6, we can get a series of IMF components (IMF1 ~ IMF9) and a residual components (res), as shown in Figure 8. Each IMF component contains the characteristic time scale of the original signal. As can be seen from Figure 8, IMF1 ~ IMF5 contain more noise, res reflects the trend of the signal. Therefore, the reconstructed signals will include IMF6 ~ IMF9 and the residual components res. The denoising result is shown in Figure 10. We can see that the signal becomes smoother, but large distortions occur in details section.

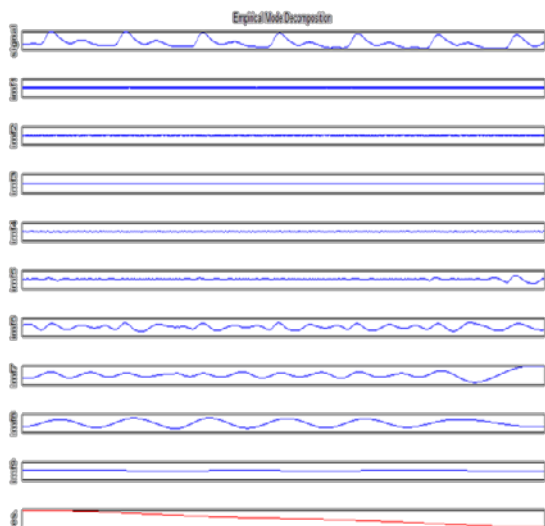


Figure 8: EMD decomposition

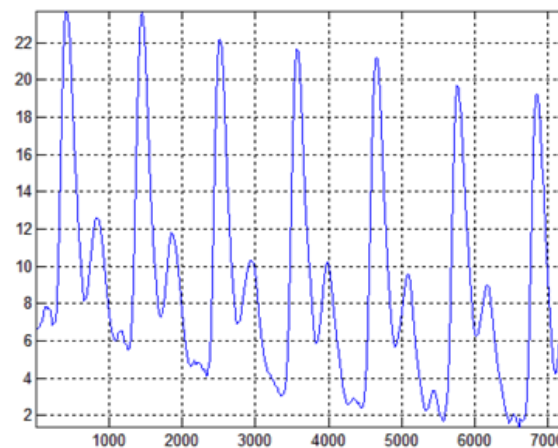


Figure 10: EMD wavelet threshold de-noising results

4).Improved EMD wavelet threshold de-noising

Compute the average of the EMD decomposed IMF1 ~ IMF9 components by Equation (12), and the results are shown in Table 1. It can be seen that CMSE4 is the minimum, so IMF1~IMF4 belong to noise, which should be denoised by the improved wavelet thresholding. When denoising, we choose $m=0.8$ and $a=2$.

The final denoised signal should be reconstructed by the denoised IMF components, untreated IMF components and the residual components res. The final denoised signal is shown in Figure 11.

3).EMD wavelet threshold de-noising

Process the pulse signal in Figure 6 by using EMD and the traditional soft threshold wavelet denoising method, where the threshold estimation is obtained according to the fixed threshold rule. The denoising results are shown in Figure 11, where most of the noise is filtered out, so the de-noised signal becomes smoother. But in the vicinity of the wave the processing is very rough, resulting in the distortion of the signal.

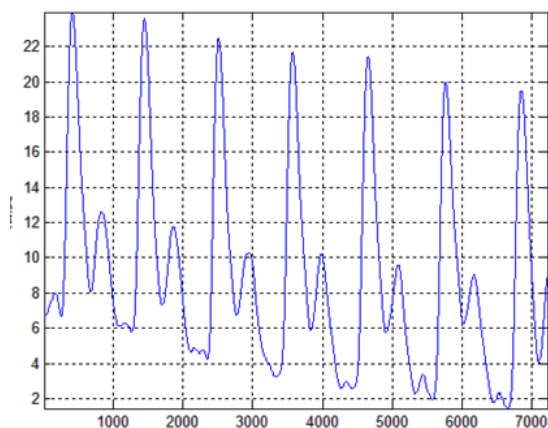


Figure 11: Improved EMD wavelet thresholding results

Table 1: Continuous MSE of IMF components

Continuous MSE	value
CMSE1	4.3160e-04
CMSE2	1.8300e-04
CMSE3	1.6439e-04
CMSE4	1.4459e-04
CMSE5	1.1836
CMSE6	7.4996
CMSE7	14.4801
CMSE8	16.7327
CMSE9	0.0721

Table 2: SNR, MSE of three de-noising methods

Method	SNR	MSE	r
Wavelet thresholding	34.9614	0.0358	0.9994
EMD	19.7616	1.1844	0.9808
EMD Wavelet thresholding	36.0048	0.0281	0.9995
Improved method	51.8292	7.3579 e-004	1.0000

From Figure 11 we can see that the reconstructed pulse signal is smooth, so the noise interference is suppressed, and the effective signal is preserved. From Table 2, it can be seen that SNR、MSE and r (cross-correlation coefficient) of the Improved method got obvious increasing than wavelet denoising and EMD denoising method. It indicates that the proposed improved method achieves better denoising effect.

V. Conclusions

This paper studies EMD and Wavelet thresholding methods. On the basis of wavelet transform and empirical mode decomposition, we put forward an improved wavelet thresholding method based on EMD. Then the human pulse signals are denoised by the improved method. Experimental results show the feasibility and superiority of the improved method.

Acknowledgement

This paper is from a scientific research program funded by science and technology bureau of Beilin district. And the authors want to thank to Gaoyuan Li and Weiyou Peng for their kind help.

References

- [1]. P.Flandrin. Empirical mode decomposition as a filter bank. *IEEE Signal Processing Letters*, 2003, 11(2): 112-114P.
- [2]. Z. H. Wu, N. E. Huang. Ensemble empirical mode decomposition: a noise assisted data analysis method. *Calverton center for Ocean-land-Atmosphere Studies*, 2003: 855-895P.
- [3]. Tan Shanwen, Qin Shuren, Tang Baoping. Filter characteristics Hilbert-Huang transform and its application [J] *Chongqing University*, 2004, 27 (2): 9-12.
- [4]. Donnbo D. Denoising by soft-thresholding[J]. *IEEE Transactions on Information Theory*, 1995, 4(3):613-615.J. S. Brown, A. Collins, and P.Duguid, "Situated Cognition and the Culture of Learning," *Educational Researcher*, vol. 18, no. 1, pp. 32-42, Jan. 1989.
- [5]. Boudraa A O, Cexus J C. EMD-based signal filtering[J]. *IEEE Transactions on Instrumentation and Measurement*, 2007, 56(6):2196-2202.
- [6]. Weaver J, Xu Y, Healy D, et al. Filtering MR images in the wavelet transform domain[J]. *Magn. Reson. Med*, 1991, 21:288-295.

- [7]. Donoho D L, Johnstone I M. Ideal spatial adaption by wavelet shrinkage [J]. Biometrika, 1994, 41(3):613-627.
- [8]. Zhang Lian, Qin Huafeng, Yu Chengbo. Wavelet thresholding algorithm [J]. Computer Engineering and Applications, 2008,44 (9): 172-174.



Shiru Zhang (used name Minrui Zhang) was born in Xian, China, on July, 1965. She received her Ph.D. degree in signal and information processing from Xidian University in China in 2005. She is with Xi'an University of Science and Technology in China, and now is a professor there and a visiting professor of National Chin-Yi University of Technology in Taiwan. She was a visiting scholar in UIUC, Illinois, USA. from May 2008 to Nov. 2009. She is a member of IEEE, a senior member of China Institute of Communications, a member of China Society of Image and Graphics, a vice general secretary of Shaanxi Signal Processing Society. Her current research interests include information hiding and digital image processing.



Xiaotong Chen was born in 1989, Beijing, China. She is a graduate student in Xi'an University of Science and Technology. Her major is electronics and communication engineering, and her research direction is pulse diagnosis.



Qingfu Sun was born in 1988, Liaocheng, Shandong, China. He is a graduate student in Xi'an University of Science and Technology. His research direction is electronics and communication engineering, and his research area is pulse diagnosis.



CaiYing Zhang was born in 1990, Wuhan, Hubei, China. Currently he is a graduate student in Xi'an University of Science and Technology. His research direction is pulse diagnosis and image processing.