

# The Coupling Relationship among Process Faults, Actuator Faults and Sensor Faults for a Typical MIMO Dynamic System

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## Abstract

For a typical MIMO (Multiple-Input Multiple-Output) nonlinear dynamic system, fault detection and isolation usually aim at process faults with an assumption that actuator faults and sensor faults do not occur at the same time, which is not always the case. This paper uses Extended State Observer for real-time process fault detection and fuzzy inference for fault isolation. It then investigates the coupling relationship among process faults, actuator faults and sensor faults, and presents how a combination of different types of faults could lead to undetected faults or false fault detection and isolation. Finally, a method to isolate actuator faults from process faults is presented. A three-tank MIMO nonlinear system is used to help illustrate the presented fault detection and isolation techniques.

## 1. Introduction

The main function of an observer, also known as estimator, is to extract information of the otherwise immeasurable variables for a vast number of applications that include feedback controls and system health monitoring or fault diagnosis. Over the past few decades, two classes of observer design have emerged. One relies on mathematical plant models to produce state estimates; the other uses available plant knowledge to estimate not only the state but also the part of the physical process that is not described in the plant model, i.e. disturbances. For the first class, however, it requires an accurate

mathematical model of the plant that is often unavailable in practice. In contrast, the second class provides practical state and disturbance estimation when significant nonlinearity and uncertainty are present in a dynamic system.

The term “fault diagnosis” generally refers to fault detection and isolation (FDI). The fault diagnosis for nonlinear dynamic systems using model-free or model-based approaches have received much attention lately [1-3]. The model-free approach relies on rich data collection to train neural networks in conjunction with the use of fuzzy inference system. Such an approach might be impractical, if not impossible, to collect rich experimental data. The model-based approach uses a linear or linearized model of the supervised system to generate a series of fault-indicating signals. In particular, the observer-based FDI methodologies have been developed along with the observer theory, and some of them have been successfully applied to industrial processes [4-6]. To deal with the nonlinearity and uncertainty of a dynamic system, nonlinear fault diagnosis has recently become an active research topic. There have been many observer-based residual-generation methods for fault diagnosis in a nonlinear dynamic system. Frank in [7] first proposed a nonlinear identity observer approach for fault diagnosis, followed by a survey on diagnostic observers [8] and a survey on robust residual generation and evaluation methods used in observer-based fault detection [9]. Later, Isermann [10] presented the status and applications of model-based fault detection and diagnosis. Observer-based fault-diagnosis was applied to robot manipulators using a mathematical technique called algebra of functions to design the nonlinear diagnostic observer [11]. Adaptive observers [12] and nonlinear robust-based observer schemes [13-14] that both developed an algorithm to adjust the gain matrix of observer to track the fault parameters of the system

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online have been applied to practical processes successfully. Additionally, a new concept of practical optimality using disturbance estimation for health monitoring has been proposed [15]. However, the common drawback of these observer-based fault diagnosis methods is the dependency on detailed knowledge of the process represented by its mathematical model.

In 2012, Lin [16] used Extended State Observer (ESO) for process fault detection of a nonlinear dynamic system assuming that the plant model is uncertain, either un-modeled or incorrectly modeled combined with unknown external disturbances. Nevertheless, the study was based on an assumption that neither actuator faults nor sensor faults occur at the same time as process faults. Zhang [17] investigated the issue of isolation of process faults and sensor faults for a class of nonlinear uncertain systems, but he did not address isolation of actuator faults and process faults perhaps due to complex coupling. This study investigated the issue of the coupling relationships among process faults, actuator faults and sensor faults, and proposed a method to isolate actuator faults from process faults. To better explain how a system behaves when combined types of faults occur at the same time, a strongly coupled MIMO three-tank dynamic system is used in this study.

This paper is organized as follows. Section II briefly describes the concept of Extended State Observer (ESO). Section III describes the fault detection and isolation schemes demonstrated by a three-tank system. Section IV describes isolation of sensor faults via the ESO, followed by isolation of process faults and actuator faults in Section V. Finally, the study is concluded in Section VI.

## 2. Extended State Observer

### 2.1 Extended State Observer Design

Consider a nonlinear dynamic system that can be described by

$$y^{(n)} = f(t, y, \dot{y}, \dots, y^{(n-1)}, d) + bu \quad (1)$$

Where  $y^{(n)}$  denotes the  $n$ th time derivative of  $y$ ,  $f$ , short for  $f(t, y, \dot{y}, \dots, y^{(n-1)}, d)$ , which is a lumped nonlinear time-varying function of the plant dynamics; and  $d$  is the unknown external disturbance;  $u$  is the system's input and  $b$  is a constant. In all physical systems,  $f$  and  $b$  are both bounded. From fault diagnosis point of view, the  $f$  can be thought of lumped unknown un-modeled or incorrectly modeled dynamics combined with the unknown external disturbances. Instead of separating un-modeled dynamics from the disturbance, the term  $f$  in its totality is to be estimated as an extended state of the system, together with the states of the system. Normally, an observer only provides the state estimation; but with what is known as Extended State Observer (ESO) [18-21], and the term  $f$  is treated as another state and estimated in real time.

Such additional information proves to be crucial for the FDI purposes, as will be shown in this paper. The ESO technique first developed by Han [18-19], however, is rather complex, and its implementation requires the adjustments or tuning of several parameters, which can be difficult and time consuming. Later, Gao [20] improved the ESO technique and made it more practical by using a particular parameterization method that reduces the number of tuning parameters to one. Such parameterized ESO has been successfully applied in many applications, particularly in the context of the Active Disturbance Rejection Control (ADRC) [21].

The main idea of ESO is to use an augmented state space model of (1) that includes  $f$  as an additional state. Thus, (1) can be represented in state space form as

$$\begin{cases} \dot{x}_1 = x_2 + bu = f + bu \\ \dot{x}_2 = \dot{f} = \eta(x, u, d, \dot{d}) \end{cases} \quad (2)$$

Where both  $f$  and  $\eta$  are assumed unknown.

Alternatively, in the case of single output (i.e.  $y=x_1$ ), Equation (2) can be written in a matrix form as

$$\begin{cases} \dot{x} = Ax + Bu + E\eta \\ y = Cx \end{cases} \quad (3)$$

Where

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} b \\ 0 \end{bmatrix}; \quad C = [1 \ 0]; \quad E = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The ESO can be expressed in a matrix form as

$$\begin{cases} \dot{z} = Az + Bu + L(y - \hat{y}) \\ \hat{y} = Cz \end{cases} \quad (4)$$

Or

$$\begin{cases} \dot{z}_1 = z_2 + l_1(x_1 - z_1) + bu \\ \dot{z}_2 = l_2(x_1 - z_1) \end{cases} \quad (5)$$

Where  $L = [l_1 \ l_2]$  is the observer gain vector which can be obtained using any known method such as the pole placement technique. When it is properly selected, the ESO provides an estimate of the state in Eq. (3) (i.e.  $z_i$  estimates  $x_i$ , where  $i = 1, 2$ ), where  $\hat{y}$  is the estimate of system output  $y$ . More specifically,  $z_1$  tracks the system output, while  $z_2$  tracks  $f$  which includes system internal dynamics and external disturbance. The choice of the observer gain vector  $L$ , originally consisted of a set of nonlinear gains [18-19], was simplified with linear gains so that it can be parameterized by solving the characteristic equation of the observer [20]. For instance, if gains are chosen as  $L = [2\omega_o \ \omega_o^2]^T$ , then the characteristic polynomial of Eq. (4) becomes

$$\lambda_0(s) = (s + \omega_o)^2 \quad (6)$$

Where  $\omega_o$  is the observer bandwidth, which needs to be tuned in practice to ensure that the ESO operates effectively, and this is a complex argument (Laplace's variable). In comparison with the original extended state observer, this is regarded as the improved extended state observer since the observer bandwidth is the only parameter needed to be tuned. The analysis of ESO was briefly given in [20]; a more elaborate account is given in [21]. For practitioners, however, perhaps it is just as interesting to see the various applications of ESO and their success in providing a practical solution in dealing with uncertainties [20, 22]. The estimation error of the ESO is described in the next section.

## 2.2 Estimation Error Convergence

In this section, the estimation error convergence is presented. Let

$$\tilde{\xi}_i(t) = x_i(t) - z_i(t), \quad i = 1, 2$$

From (2) and (4), the observer estimation error for states  $x_1$  and  $x_2$  can be described as

$$\begin{cases} \dot{\tilde{\xi}}_1 = \tilde{\xi}_2 - l_1 \tilde{\xi}_1 \\ \dot{\tilde{\xi}}_2 = \eta - l_2 \tilde{\xi}_1 \end{cases} \quad (7)$$

Now let us scale down the observer estimation error

$$\tilde{\xi}_i(t) \text{ by } \omega_o^{i-1}, \text{ i.e., let}$$

$$\varepsilon_i(t) = \frac{\tilde{\xi}_i(t)}{\omega_o^{i-1}}, \quad i = 1, 2.$$

Then, (7) can be written as

$$\dot{\varepsilon} = \omega_o A_e \varepsilon + B_e \frac{\eta(x, u, d, \dot{d})}{\omega_o} \quad (8)$$

Where

$$A_e = \begin{bmatrix} -2 & 1 \\ -1 & 0 \end{bmatrix}, \quad B_e = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Here  $A$  is Hurwitz for  $L = [l_1 \ l_2] = [2\omega_o \ \omega_o^2]$ .

**Theorem 1:** Assuming  $\eta(x, u, d, \dot{d})$  is bounded, then there exists a constant  $\sigma_i > 0$  and a finite time  $T_1 > 0$  such that  $|\tilde{\xi}_i(t)| \leq \sigma_i, \quad i = 1, 2, \forall t \geq T_1 > 0$  and  $\omega_o > 0$ .

Note that

$$\sigma_i = O\left(\frac{1}{\omega_o^k}\right)$$

for some positive integer  $k$ . The boundedness of  $\eta(x, u, d, \dot{d})$  (i.e.  $\dot{f}$ ) means that the rate of change of the combined effect of internal dynamics and external disturbances is finite, which leads to an assumption that the combined effect and the control input are continuous. Here  $\eta$  is essentially the derivative of acceleration. In a typical motion system,  $\eta$  being bounded means that the force applied to the body

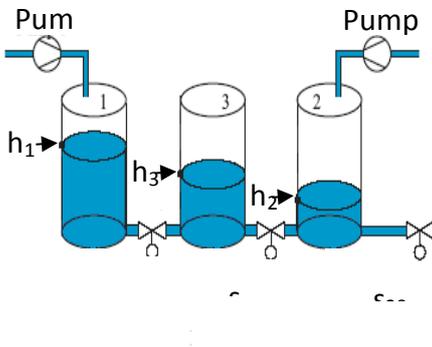
does not change infinitely within a very short period of time. In other words, the jerk (i.e. time derivative of acceleration) is finite. This is a reasonable assumption for a typical motion.

**Remark 1:** The mathematical proof of Theorem 1 can be found in [16] that, in the absence of the plant model, the estimation error of the ESO as described in (4) is bounded and its upper-bound monotonously decreases with the observer bandwidth.

As long as the bandwidth is sufficiently large, the ESO can be used to estimate the state as well as the extended state  $f$  which includes system internal dynamics and external disturbance.

### 3. Case Study: Three-Tank System

To illustrate how the presented ESO can be used to track a nonlinear dynamic system. A three-tank nonlinear dynamic system [3] as shown in Fig. 1 is used here as a case study. The system consists of three tanks ( $T_1$ ,  $T_2$  and  $T_3$ ) that are connected by three pipes. The system has two controlled inputs (pump flow rates), three measurable outputs  $h_1$ ,  $h_2$  and  $h_3$  (water levels), and three possible faults (pipe blockages). It is, indeed, a strongly coupled MIMO system.



**Figure 1: Schematic Diagram of the Three-Tank System**

Using the Torricelli's law, the following three dynamic system equations can be obtained

$$\begin{cases} A_T \frac{dh_1}{dt} = -s_{13}a_1 \text{sign}(h_1 - h_3) \sqrt{2g|h_1 - h_3|} + Q_1 \\ A_T \frac{dh_2}{dt} = s_{32}a_3 \text{sign}(h_3 - h_2) \sqrt{2g|h_3 - h_2|} - s_{20}a_2 \sqrt{2gh_2} + Q_2 \\ A_T \frac{dh_3}{dt} = s_{13}a_1 \text{sign}(h_1 - h_3) \sqrt{2g|h_1 - h_3|} - s_{32}a_3 \text{sign}(h_3 - h_2) \sqrt{2g|h_3 - h_2|} \end{cases} \quad (9)$$

Where

$A_T$  is the circular cross-sectional area of each tank (assumed same for all);

$a_1, a_2, a_3$ : the circular cross-section area of each pipe;

$s_{13}, s_{32}, s_{20}$ : pipe flux coefficient;

$Q_1, Q_2$ : pump's flow rate;

$h_1, h_2$  and  $h_3$  denote the water level of tanks  $T_1, T_2$  and  $T_3$ , respectively.

The flux coefficient is between 0 and 1, where "0" and "1" represent complete blockage and no blockage, respectively. Equation (9) can be rewritten as

$$\begin{cases} \dot{h}_1 = f_1 + \frac{1}{A_T} Q_1 \\ \dot{h}_2 = f_2 + \frac{1}{A_T} Q_2 \\ \dot{h}_3 = f_3 \end{cases} \quad (10)$$

Where

$$f_1 = -\frac{1}{A_T} [s_{13}a_1 \text{sign}(h_1 - h_3) \sqrt{2g|h_1 - h_3|}]$$

$$f_2 = \frac{1}{A_T} [s_{32}a_3 \text{sign}(h_3 - h_2) \sqrt{2g|h_3 - h_2|} - s_{20}a_2 \sqrt{2gh_2}]$$

$$f_3 = \frac{1}{A_T} [s_{13}a_1 \text{sign}(h_1 - h_3) \sqrt{2g|h_1 - h_3|} - s_{32}a_3 \text{sign}(h_3 - h_2) \sqrt{2g|h_3 - h_2|}]$$

Let  $y(t)$  and  $u(t)$  be the system's output and input vector, respectively,

$$y(t) = [h_1 \ h_2 \ h_3]^T; \quad u(t) = [Q_1 \ Q_2 \ 0]^T \quad (11)$$

Where  $h_1, h_2$  and  $h_3$  denote the water level (m) of tanks  $T_1, T_2$  and  $T_3$ , respectively, and  $Q_1$  and  $Q_2$  denote the flow rate ( $\text{m}^3/\text{sec}$ ) of pumps 1 and 2, respectively. Essentially, the water levels are the system output variables, and the flow rates are the system input variables. The three possible blockage faults are process faults; the other two types of faults are actuator faults in the two pumps and sensor faults in measuring the output variables (the water levels in this case). Combining (9) and (10) gives

$$\dot{y}(t) = f + b_0 u(t) \quad (12)$$

Where

$$b_o = \frac{1}{A_T} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad f = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}$$

The  $f_1$ ,  $f_2$  and  $f_3$  are called the Generalized System Dynamics of tank  $T_1$ ,  $T_2$  and  $T_3$ , respectively, and  $u(t)$  is the system's inputs. Note that the constant  $b_o$  can be determined by the system, which in this case, is simply the reciprocal of the tank's area.

Equation (12) can be represented in state space form as:

$$\begin{cases} \dot{x}_1 = x_2 + b_o u \\ \dot{x}_2 = v \\ y = x_1 \end{cases} \quad (13)$$

Where  $u(t) = [Q_1 \ Q_2 \ 0]^T$  is the system input,  $y = x_1 = [h_1 \ h_2 \ h_3]^T$  is the system output,  $x_2 = f = [f_1 \ f_2 \ f_3]^T$  is an augmented state, and  $v$  is the time derivative of  $f$ . Rewriting (13) in matrix form gives

$$\begin{cases} \dot{x} = Ax + Bu + Dv \\ y = Cx \end{cases} \quad (14)$$

Where

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{6 \times 1}, \quad A = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}_{6 \times 6}, \quad B = \begin{bmatrix} b_o \\ 0 \end{bmatrix}_{6 \times 3}, \quad D = \begin{bmatrix} 0 \\ I \end{bmatrix}_{6 \times 3}$$

$$C = [1 \ 1 \ 1 \ 0 \ 0 \ 0]$$

and  $I$  is a three-by-three identity matrix. Note that the expression for  $C$  in (14) is for three outputs, while that for  $C$  in (3) is for single output.

Employing the ESO design (using Eqs (2) - (6)), denoting  $y$  as the measured or actual output,  $\hat{y}$  as the estimated output, and incorporating the difference between the two outputs, the ESO in (13) can be rewritten as

$$\begin{cases} \dot{z}_1 = z_2 + l_1(x_1 - z_1) + b_o u \\ \dot{z}_2 = l_2(x_1 - z_1) \end{cases} \quad (15)$$

The state space observer can be constructed as

$$\begin{cases} \dot{z} = Az + Bu + L(x_1 - z_1) \\ \hat{y} = Cz \end{cases} \quad (16)$$

Where

$$z = [z_1 \ z_2]^T \text{ (i.e. } z_1 = [z_{11} \ z_{12} \ z_{13}]^T; z_2 = [z_{21} \ z_{22} \ z_{23}]^T \text{)}$$

Equation (9) shows that a three-tank system consists of three simultaneous first-order differential equations. Thus, the observer gain matrix,  $L$  can be expressed as

$$L = \begin{bmatrix} 2\omega_o & 0 & 0 \\ 0 & 2\omega_o & 0 \\ 0 & 0 & 2\omega_o \\ \omega_o^2 & 0 & 0 \\ 0 & \omega_o^2 & 0 \\ 0 & 0 & \omega_o^2 \end{bmatrix} \quad (17)$$

With a chosen bandwidth  $\omega_o$ , the  $z$  vector can be used to estimate the system outputs and the system dynamics in real time. As stated in the *Theorem 1*, the ESO's estimation error is upper-bounded and monotonously decreases with the bandwidth. Thus, with a sufficiently large bandwidth and as time proceeds,  $z_1$  quickly approaches  $y$  (i.e.  $h_1$ ,  $h_2$  and  $h_3$ ), and  $z_2$  approaches  $f$  (i.e.  $f_1$ ,  $f_2$  and  $f_3$ ). In other words,  $z_1$  tracks the system's outputs, and  $z_2$  tracks the un-modeled system dynamics combined with external disturbance. More specifically, as stated in (16)  $z_1 = [z_{13} \ z_{11} \ z_{12}]^T$  estimates the state variables  $x_1$  (i.e. the water level  $h_1$ ,  $h_2$  and  $h_3$ ), and  $z_2 = [z_{21} \ z_{22} \ z_{23}]^T$  estimates the extended state  $f$  (i.e.  $f_1$ ,  $f_2$  and  $f_3$ ). In other words,

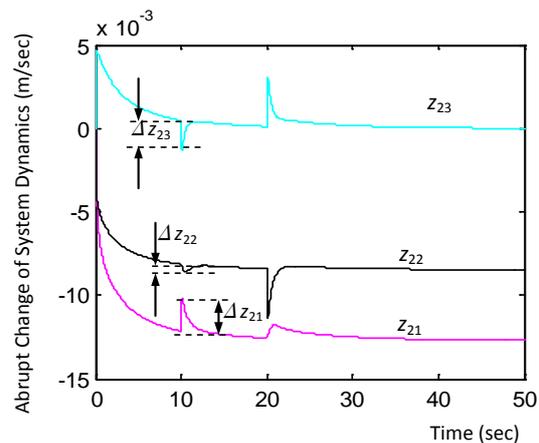
$$\begin{cases} z_{11} = \hat{h}_1 \rightarrow h_1; & z_{12} = \hat{h}_2 \rightarrow h_2; & z_{13} = \hat{h}_3 \rightarrow h_3 \\ z_{21} \rightarrow f_1; & z_{22} \rightarrow f_2; & z_{23} \rightarrow f_3 \end{cases} \quad (18)$$

### 3.1 Fault Detection by Means of the ESO

For the given three-tank system, the process faults are the pipe blockage faults in  $s_{13}$ ,  $s_{32}$  and  $s_{20}$  as shown in Fig. 1. Traditionally, faults are considered detected when the outputs exceed the expected values by a preset tolerance. This approach, however, has some drawbacks in open-loop and closed-loop

controls. When using the ESO for closed-loop control, observing the system's output does not provide useful information about the health of the system because the controller tries to augment the inputs in an effort to stabilize the system. Thus, the health does not surface until the system finally collapses. Using the ESO for open-loop control also encounters a problem before the system reaches its steady states. In other words, an abrupt change on the system output does not necessarily mean the system is becoming faulty. Thus, solely relying on monitoring the system output could trigger a false alarm or miss detection of possible faults.

It is worthwhile to note that the ESO's unique feature is its ability to estimate the general system dynamics (i.e. the un-modeled system dynamics and unknown external disturbance) in real time, which provides crucial information for the presented fault detection technique. Our study found that the system outputs and the general system dynamics both exhibit abrupt changes as soon as a fault occurs. However, the rate of change on the general system dynamics is more profound. Furthermore, the system outputs potentially contain process faults (such as the pipe blockage faults), actuator faults (such as the actuating faults in the pumps), sensor faults or any combination of the faults. For this reason, our proposed fault detection scheme is primarily based on the general system dynamics  $f$ . More specifically, a fault is considered detected when the rate of change of general system dynamics,  $\Delta f/f$ , exceeds the predetermined threshold value. An example of multiple faults detection is shown below in Fig. 2 where  $\Delta z_{21}$ ,  $\Delta z_{22}$  and  $\Delta z_{23}$  correspond to  $\Delta f_1$ ,  $\Delta f_2$  and  $\Delta f_3$ , respectively.



**Figure 2: Detection of multiple faults ( $s_{13} = 0.8$  at  $t = 10$  sec. and  $s_{32} = 0.6$  at  $t = 20$  sec.)**

### 3.2 Fault Isolation by Means of the ESO

In addition to monitoring the system outputs, the system dynamics used for fault detection can be used for fault isolation. Referring to Fig. 2 when the first fault occurs at  $t=10$  sec., if  $\Delta z_{21}$  (the ESO's estimated  $\Delta f_1$ ) is positive,  $\Delta z_{22}$  (the ESO's estimated  $\Delta f_2$ ) is negative, and  $\Delta z_{23}$  (the ESO's estimated  $\Delta f_3$ ) is negative, then a blockage fault between tanks 1 and 3 (i.e.  $s_{13}$ ) likely has occurred. When the second fault occurs at  $t=20$  sec., if  $\Delta z_{21}$  is positive,  $\Delta z_{22}$  is negative and  $\Delta z_{23}$  is positive, then a blockage fault between tanks 3 and 2 (i.e.  $s_{32}$ ) likely has occurred. The observations suggest that some intuitive logic, better known as fuzzy logic, can be employed to isolate the faults.

A fuzzy inference system (FIS) consists of input membership functions, output membership functions and the if-then fuzzy logic rules. Among them, constructing the proper input membership functions is critical, and can be most difficult if there is no prior knowledge about how input data are distributed. The best way to determine data distribution is through the use of histograms. The FIS's inputs variables are  $\Delta z_{21}$ ,  $\Delta z_{22}$  and  $\Delta z_{23}$  which are normalized to the range of  $[-1, 1]$ . The output variables are the degree of fault for  $s_{13}$ ,  $s_{32}$ ,  $s_{20}$ , which are normalized to the range of  $[0, 1]$ , where "0" represents complete fault, and "1" represents no fault.

## 4. Isolation of Sensor Faults

The occurrence of a sensor fault typically causes a bias to occur in the measurements of the affected sensor. The sensor faults investigated here were introduced via an instantaneous numerical offset at a specific time after reaching the system's steady state.

The unique behavior of the sensor fault that distinguishes it from the actuator and process faults can be mathematically explained via the matrix algebra of the ESO:

$$\begin{aligned} \dot{z}_1 &= z_2 + l_1(x_1 - z_1) + bu \\ \dot{z}_2 &= l_2(x_1 - z_1) \end{aligned} \quad (19)$$

For multiple output systems with  $n$  variables, the variables  $x_1$ ,  $z_1$  and  $z_2$  are column vectors with  $n$  elements. Correspondingly, the observer gains  $l_1$  and  $l_2$  are  $n$  by  $n$  diagonal matrices where each non-zero element represents the gain corresponding to a specific element of  $(x_1 - z_1)$ .

In the event of a sensor fault, the immediate change in one of the measured variables causes one of the elements of  $(x_1 - z_1)$  to become nonzero (as the ESO values previously matched the measurements, all elements in  $(x_1 - z_1)$  were zero). After multiplication by the diagonal gain matrix, the resulting vector also contains only one nonzero element, with the same index. More specifically, a remark can be made.

**Remark 2:** if the  $i$ -th sensor experiences a fault, only the  $i$ -th elements of the observable state variable and the extended state variables are affected.

A process or actuator fault usually affects values of more than one observable variables and extended state variables as calculated by the ESO. Isolation of process faults from actuator faults exhibits complex coupling effects for a typical MIMO system, such as the three-tank system. Isolation of process faults and actuator faults are discussed in the next section.

## 5. Isolation of Process Faults and Actuator Faults

### 5.1 Characteristics of Process and Actuator Faults

In order to distinguish actuator and process faults, the way they affect the convergence of extended state variables calculated by the ESO must be examined. Process faults alter the dynamics of the system, and thus the final steady state values of the state variables (in this case the tank heights) are different. Actuator faults cause unexpected discrepancies in the system input and thus also result in different steady state variable values. However, due to the calculation method of the ESO, the steady state values of the extended state variables of  $z_2$  are also affected. At steady state, all time derivatives become zero and the ESO's estimated values match the measured values, which cause (19) to reduce to the following:

$$0 = z_2 + l_1(0) + bu \quad (\text{i.e. } z_2 = -bu)$$

In the case of an actuator fault, values in vector  $u$  will be affected by the fault and deviate from the designed or expected theoretical values. However, the ESO will not know this and will still use the theoretical values. This will cause the elements of  $z_2$  to converge to incorrect values, which do not match the steady state values of corresponding functions  $f_1, f_2, \dots, f_n$ . Without the ability to compare  $z_2$  to  $f$  (as in most situations,  $f$  that represents plant dynamics must be assumed unknown), this discrepancy cannot be observed, and thus the only distinguishing characteristic of actuator faults is unobservable.

Recall that a general nonlinear system that can be modeled by the ESO is expressed in the following:

$$y^{(n)} = f + bu$$

Where  $y^{(n)}$  denotes the  $n$ th time derivative of  $y$ ,  $f$  is essentially the extended state representing the lumped nonlinear time-varying function of the plant dynamics and external disturbances,  $u$  is the system input and  $b$  is a constant which is related to the physical model.

The occurrence of a process fault causes a change in ' $f$ ' whereas an actuator fault causes a change in ' $u$ '. However, either type of fault results in a net change to the same variable,  $y^{(n)}$  which updates the system state. This makes it possible for both types of faults to produce very similar behavior in the system state and extended state variables in the case of a single fault, or cause the appearance of an undisturbed system with simultaneous actuator and process faults. Mathematically this fault ambiguity can be represented by  $\epsilon$ :

$$y^{(n)} = f + bu + \epsilon \quad (20)$$

For actuator faults or process faults,  $\epsilon$  represents a change in either the input  $b(u + \Delta u)$  or the system dynamics  $f + \Delta f$ . The fact that  $\epsilon$  could be either  $b\Delta u$  or  $\Delta f$  allows for either type of a single fault to produce similar states if  $b\Delta u = \Delta f$  or simultaneous faults resulting in a seemingly undisturbed state if  $b\Delta u = -\Delta f$ .

**Remark 3:** For a typical MIMO nonlinear system, actuator faults, in general, cannot be isolated from process faults unless one or more additional sensor measurements are made.

## 5.2 Utilizing an Outflow Sensor to Isolate Actuator Faults

As discussed earlier, at least a sensor measurement must be added in order to resolve the ambiguity between process and actuator faults. Taking the presented three-tank system as an example, one can add an outflow sensor at the very end of the pipe (right outside the right hand side of tank 2) to measure the system's net outflow. At steady state, conservation of mass dictates that the outflow of the three-tank system must be equal to the sum of the inputs  $Q_1 + Q_2$  (i.e. pump flow rates). The theoretical steady state value of this quantity is only dependent on system specifications, and the actual outflow must always converge to the theoretical value except in the event of an actuator fault. As no non-actuator fault can affect  $Q_1$  or  $Q_2$ , measuring the net outflow allows for a means to isolate actuator faults. This concept will work only if the added outflow sensor itself is not faulty. In fact, there is a simple way to determine if the outflow sensor itself is faulty. After the system

reaches the steady state, if the net outflow does not equal  $Q_1 + Q_2$ , but there is no noticeable disturbance in the observable state and the extended state variables, then it is likely the outflow sensor is faulty. However, if the net outflow does not equal  $Q_1 + Q_2$ , and there is noticeable disturbance in those variables, then there exists a fault in actuator 1 or 2. The only means to isolate one actuator fault from the other is to directly measure each actuator's output. Actuator faults should be closely monitored because it affects the isolability of process faults.

## 6. Conclusion

The detection and isolation of process faults by means of extended state observer (ESO) and fuzzy inference have been presented. This study was conducted via the computer simulation in which the equations for the three-tank system were used to calculate the theoretical values. To simulate un-modeled dynamics, 5% to 10% of external disturbance was introduced. The ESO was found capable of filtering system noises and correctly detecting process faults even when the system was not correctly modeled (when using  $b_o = 635$  as supposed to the exact value of 127). However, in reality, process faults could be accompanied by sensor faults and/or actuator faults. The coupling relationships among the three types of faults were investigated. Among them, sensor faults can be easily detected and isolated. For a strongly coupled MIMO nonlinear system, combination of process faults and actuator faults exhibits complex coupling effects because these two types of faults affect the values of both observable state and extended state variables. For the given three-tank system, a method for isolating actuator faults from process faults was presented. Future work includes developing a general methodology to isolate actuator faults from process faults for a strongly coupled nonlinear system without having to add additional measurements.

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