Using Modified Particle Swarm Optimization to Design a Controller by Time-Domain Objective Functions

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Abstract

A large proportion of industrial systems are represented by linear time-invariant transfer functions. The proportional-integral -derivative (PID) controller is one of the most widely used functions. The lead-lag controller is a more practical alternative. Lead-lag-like controller can be lead-lead, lag-lead or lag-lag controller. This paper focused on the design of lead-lag-like controller by optimization of the time-domain objective function. A modified particle swarm optimization (MPSO) algorithm is chosen to find the optimal solutions. MPSO tries to force all the particles to search the optimum exhaustedly. The proposed objective function includes time-domain specifications, including the delay time, rise time, first peak time, maximum peak time, maximum overshoot, maximum undershoot, setting time and steady state error. Designers can make trade-offs in a variety of specifications. As long as the plant could be modeled as a linear time-invariant transfer function, the suggested method would design the lead-lag-like controller capable of approaching the desired specifications. Computer simulations show that the performance can be fully met or very close to desired.

Keywords: Lead-lag controller, PID controller, PSO

1. Introduction

Most industrial systems can be represented by a linear time-invariant transfer function. Lead-lag controllers provide a more practical alternative. Quite a few typical methods have been proposed for tuning the PID controller parameters over the years [1-5]. More practical alternatives are lead-lag controllers. The design the lead-lag controller has been studied [6-8]. Particle swarm optimization (PSO) is one of the meta-heuristic techniques developed by Kennedy and Eberhart [9, 10]. Ou and Lin proposed a method based on GA and PSO to design the PID controller, and then compared the results [11]. Horng used greedy PSO to design lead-lag controller [12]. Modified particle swarm is inspired by the literature [13]. The center particle has capacity to get good solutions. Hence, the concept of center particle is included in velocity update formula of a modified particle swarm algorithm.

In this paper, modified particle swarm optimization that uses a time-domain objective function to design a controller is proposed. If the plant could be modeled as a linear time-invariant transfer function, the proposed method can design a controller that approaches or meets the time-domain specifications. The objective function includes eight specifications, including delay time, rise time, first peak time, maximum peak time, percent maximum overshoot, percent maximum undershoot, setting time and steady state error.

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2. Time-Domain Specifications

The time response of a control system is divided into two parts: the transient response and the steady-state response. Let **y**(t) denote the time response of a continuous system; then, in general, it can be written as

$$y(t) = y_t(t) + y_{ss}(t) .$$

Where $y_t(t)$ denotes the transient response, and $y_{ss}(t)$ means the steady-state response.

In control systems, transient response is defined as the part of the time response that goes to zero as time becomes very large. Therefore, $y_t(t)$ has the property

$$\lim_{t\to\infty} y_t(t) = 0 .$$

The study of a control system in the time domain basically involves the evaluation of the transient and the steady-state responses of the system. In the design problem, specifications are usually given in terms of the transient and the steady-state performances.

The transient response is certainly important because it is a significant part of the dynamic behavior of the system; and the difference between the output response and the input response, before the steady state is reached, must be strictly controlled.

The steady-state response is basically the part of the total response which remains after the transient has died out.

Therefore, controllers are designed so that the specifications are all met by the designed system.

2.1 Transient Response of Step Input

For a general system, the following specifications will be considered:

1).Delay time. The delay time T_d defined as the time required for the step response to reach 50% of its final value $y_{ss}(t)$.

2).Rise time. The rise time T_r is defined as the time required for the step response to rise from 10 to 90% of its final value.

3).First peak time, T_p, Is the time to reach the first peak.

4).Maximum peak time, T_m, is the time to reach the maximum peak.

5).Percentage maximum overshoot, $\%OS$, is the amount that the maximum waveform overshoots the steady state value at the maximum peak time, expressed as a percentage of the steady-state value.

6).Percentage maximum overshoot, $\%US$, is defined as

$$y_{us} = \min(y(t)),\ t \ge T_p,$$

$$\%US = \begin{cases} \dfrac{(y_{ss} - y_{us})}{y_{ss}}, & \text{if } y_{us} < y_{ss}, \\ 0, & \text{if } y_{us} \ge y_{ss}. \end{cases}$$

7).Settling time, T_s, is the time required for the step response to decrease and stay within a specified ±2% of its final value.

Notice that, for second order system, the first peak time is always the maximum peak time. However, for the general system, they are not the same. For simplicity, in all of the examples in this article, these two values are set to the same.

2.2 Steady-State Error

One of the purposes of most control systems is that the system output response follows a particular reference signal accurately in the steady state. The error of the system may be defined as

$$e(t) = r(t) - y(t),$$

Where $r(t)$ is the input signal that the output $y(t)$ is to follow. The steady-state error is defined as

$$E_{ss}(t) = \lim_{t\to\infty} e(t)$$

In unity negative feedback systems as shown in Figure 1, it is assumed that the closed-loop system is stable, which yields:

$$E_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+G(s)}. \quad (1)$$

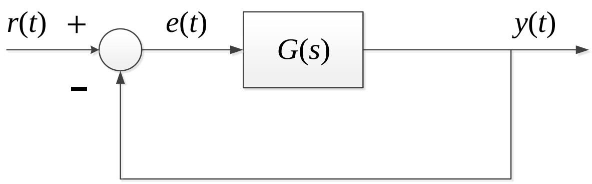

Figure 1: Unity negative feedback systems.

Three test signals are used to establish specifications for a control system's steady-state error characteristics.

1).**Step-Function Input**: The step-function input denotes an instantaneous change in the reference input. The mathematical description of a step function or magnitude R is

$$r(t) = R,\ t \ge 0$$
$$= 0,\ t < 0$$

Where R is a real constant. That is

$$r(t) = Ru_s(t)$$

Where $u_s(t)$ is the unit-step function. The Laplace transform of $r(t)$ is $R(s) = \frac{R}{s}$.

2).**Ramp-Function Input**: The ramp function is a signal that changes regularly with time. Mathematically, a ramp function is represented by

$$r(t) = Rtu_s(t)$$

Where R is a real constant. The Laplace transform of $r(t)$ is $R(s) = \frac{R}{s^2}$.

3).**Parabolic-Function Input**: The parabolic function represents a signal that is one order faster than the ramp function. Mathematically, it is represented as

$$r(t) = \frac{Rt^2}{2} u_s(t)$$

Where R is a real constant. The Laplace transform of $r(t)$ is $R(s) = \frac{R}{s^3}$

Now let us consider the effects of the types of inputs on the steady-state error. In the study, only the step, ramp, and parabolic inputs are considered.

(a) Step-Function Input.

$$E_{ss} = \frac{1}{1 + \lim_{s\to 0} G(s)} = \frac{1}{1+K_p}, \quad K_p = \lim_{s\to 0} G(s). \quad (2)$$

(b)Ramp-Function Input.

$$E_{ss} = \frac{1}{\lim_{s\to 0} sG(s)} = \frac{1}{K_v}, \quad K_v = \lim_{s\to 0} sG(s). \tag{3}$$

(c)Parabolic-Function input.

$$E_{ss} = \frac{1}{\lim_{s\to 0} s^2G(s)} = \frac{1}{K_a}, \quad K_a = \lim_{s\to 0} s^2G(s). \tag{4}$$

The relation between the input, steady-state error, and system type is shown in Table 1.

Table 1: Relationships between input, system type and E_{ss}

Input	Type 0 System	Type 1 System	Type 2 System
Step	$E_{ss} = \frac{1}{1+K_p}$	$K_p = \infty, E_{ss} = 0$	$K_p = \infty, E_{ss} = 0$
Ramp	$K_v = 0, E_{ss} = \infty$	$E_{ss} = \frac{1}{K_v}$	$K_v = \infty, E_{ss} = 0$
Parabola	$K_a = 0, E_{ss} = \infty$	$K_a = 0, E_{ss} = \infty$	$E_{ss} = \frac{1}{K_a}$

3. Objective Functions

The main purpose of most control systems is that the system output response follows a specific reference signal accurately all the time. It is well known that the time response of a control system is divided into two parts: the transient response and the steady-state response. In the real world, the steady state of the output response rarely agrees exactly with the reference. Therefore, steady-state errors in control systems are practically unavoidable. In a design problem, one of the objectives is to keep the steady-state error to a minimum, or below a certain tolerable value. Furthermore, the transient response must satisfy a certain set of specifications. The objective functions proposed is aimed at satisfying all the specifications of transient response and steady-state response listed above. For practical purposes, all of the time domain specifications have tolerance, i.e., lower bound (lb) and upper bound (ub).

First, define deviation ratio (DR)

$$\mathrm{DR}(TDS) = f(\mathbf{x}|TDS : lb,ub)$$
$$= \begin{cases} 0, & \text{if } 0 \le lb \le f(\mathbf{x}|TDS) \le ub \\ \dfrac{f(\mathbf{x}|TDS) - ub}{ub}, & \text{if } f(\mathbf{x}|TDS) > ub \\ \dfrac{lb - f(\mathbf{x}|TDS)}{lb}, & \text{if } f(\mathbf{x}|TDS) < lb \end{cases} \tag{5}$$

Where TDS is the time-domain specification, i.e. rise time, first peak time, maximum peak time, etc. Deviation ratio represents the measurement of the difference of the actual response to desired response.

The proposed objective function is

$$TDOF = (w_1\mathrm{DR}(T_d) + w_2\mathrm{DR}(T_r) + w_3\mathrm{DR}(T_p) + w_4\mathrm{DR}(T_m)$$
$$+ w_5\mathrm{DR}(OS\%) + w_6\mathrm{DR}(US\%) + w_7\mathrm{DR}(T_s) + w_8\mathrm{DR}(E_{ss}))/TW \tag{6}$$

Where $TW = \sum_{i=1}^{8} w_i$. In equation (6), w_i represent weights reflecting the relative importance of the corresponding term. That is, the foregoing objective function is weighted deviation ratio of time-domain specifications.

4. Lead-Lag-Like Controller

The transfer function of proposed controller can be written as:

$$G_c(s) = K\left(\frac{T_1 s+1}{\alpha T_1 s+1}\right)\cdot\left(\frac{T_2 s+1}{\beta T_2 s+1}\right),$$

Where $K > 0, \alpha > 0, T_1 > 0, \beta > 0, T_2 > 0.$ If $\alpha < 1, \beta < 0$, the controller is lead-lead (or two-stage phase-lead) controller. If $\alpha > 1, \beta < 0$, the controller is lead-lag (or lag-lead) controller. Moreover, if $\alpha > 1, \beta > 0$, the controller is lag-lag (two-stage phase-lag) controller.

5. Particle Swarm Optimization

Particle swarm optimization (PSO) is meta-heuristic techniques developed by Kennedy and Eberhart [9, 10].

$$V_i^d(t+1) = wV_i^d(t) + c_1r_1^d(X_{pbest}^d - X_i^d) + c_2r_2^d(X_{gbest}^d - X_i^d) \tag{7}$$

$$X_i^d(t+1) = X_i^d(t) + V_i^d(t+1) \tag{8}$$

In the above procedures, w is inertia weight and set to 0.7. Furthermore, c_1, c_2 are constants known as acceleration coefficients, and their values are 1.7 according to past experiences.

The center particle has capacity to get good solutions. More importantly, the center particle has more opportunities to become the best global swarm. Hence it can guide the whole swarm to favorable regions and accelerate convergence. Modified particle swarm (MPSO) is inspired by the literature [13]. The center particle has capacity to develop good solutions. Therefore, the concept of a center particle is included in velocity update formula. Notice that, in

Equation 7, if $V_i^d(t)=0$, and $X_i^d = X_{pbest}^d = X_{gbest}^d$, the particle will not move anymore. To overcome this drawback, the formula of modified particle swarm optimization is as follows:

$$V_i(t+1) = wV_i\ (t) + c_1 r_1\ (X_{pbest} - X_i\ ) + c_2 r_2\ (X_{gcbest} - X_i\ ) \qquad (9)$$

$$X_i\ (t+1) = X_i\ (t) + V_i\ (t+1) \qquad (10)$$

Where,

$$X_{gcbest} = (1-\alpha) X_{gbest} + \alpha X_{mpbest},$$

$$X_{mpbest} = \sum_{i=1}^{n} X_{pbest} = \text{mean}(X_{pbest}).$$

$$\alpha = \mu \times \frac{(\text{iterations -1})}{\text{maximum iterations}}$$

The formulae try to force all the particles to search the optimum solution until the average of all the individual optimal value is equal to the global optimal value. For that reason, MPSO has the ability to approach better solutions. In the paper, μ is equal to 0.1 according to previous experiences.

6. Design Procedure

The main purposes of most control systems is that the system output response follows a specific reference signal accurately. In the following design, one of the objectives is to keep the steady-state error below a certain tolerable value and satisfy the specifications of transient response. The proposed objective function is listed in Equation 6. Hence, the problem of controller design now becomes the optimizing the proposed objective function. To design the controller, you must know the transfer function of the system and the type of system. First, the desired interval of steady state error is determined, then the gain *K* of lead-lag-like controller can be calculated using Equation 2 or Equation 3, or Equation 4. Next, the parameters of a controller, i.e. α, T_1, β and T_2 are set to the interval [0.01, 100]. To save computing time, the initial particles in this article are screened, so that they will let the closed loop stable. That is, the initial values of the controller randomly generated must be satisfied Routh-Hurwitz criterion [1, 2]. This design of the controller is divided into the following eight steps.

1).Determine desired peak time and maximum overshoot.
2).Draw sketches of response, and estimate T_d, T_r, and T_s.
3).Determine the upper limits of *%OS*, *%US*, and E_{ss}.
4).Set the maximum iterations to 1000.
5).Randomly generated 30 initial particle.
6)Establish lower and upper bounds of controller parameters.
7).Run MPSO 30 times. Let X_{gbest}^i be elite initial $X_{initial}^i$.
8).Run MPSO one more time to get final parameters.

7. Illustrative Examples

Three numerical examples for a unity feedback system are provided. First example, the plant is type 0 system, second example the plant is type 1 system, and in final example the plant will cause the closed loop unstable.

Example 1: Suppose the plant transfer function is given by

$$G_p = \frac{140}{s^2 + 14s + 40}.$$

The step response of uncompensated system is shown in Figure 2. Here, the peak time is 0.2763 seconds, the percentage overshoot is 14.64%, and the steady-state error e_{ss} is equal to 0.2222.

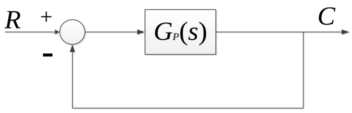

Figure 2: Unity feedback system without controller

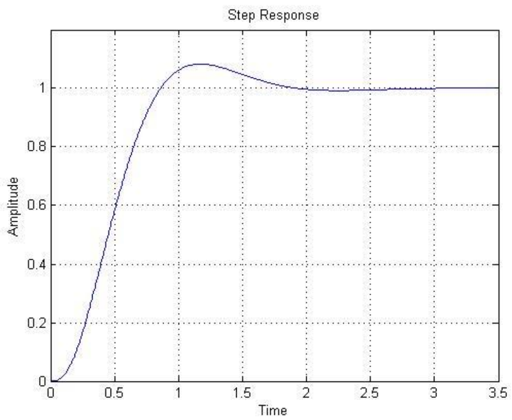

Figure 3: Uncompensated system in Example 1

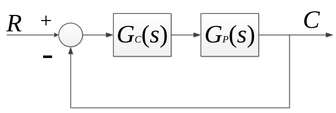

Figure 4: A unity feedback system with a controller

Clearly, the step response of a uncompensated system is not good. Hence, the controller is needed to improve the transient response as in Fig 3. Assume that both the first peak time and maximum peak time are 0.1 seconds, and maximum overshoot is 3%. According to peak time and maximum overshoot, draw sketches of response, and estimating other specifications, for example delay time, setting time etc. For some desired specifications, i.e. T_P, T_m, T_d, T_r, and T_s , give some tolerance, such as one percent. For the others, such as the maximum overshoot, maximum undershoot, and maximum steady state error, only the upper limits are given. Finally, the desired values and deviation ratios are listed in Table 2. The plant in Example 1 is type 0 system. Then, the desired interval of steady state error is determined $0.005 \le E_{ss} \le 0.022$, the gain K of lead-lag-like controller can be calculated using equation 2, i.e. $4.7952 \le K \le 5.8726$. The parameters of controller, i.e. α, T_1, β and T_2 are set to the interval [0.01, 100].

Then the MPSO is run 30 times to get elite group and use the elite group as initial particle to run MPSO one more time. At that moment, the design procedure is completed, and the controller parameters are K= 72.9705, α = 0.1699, T_1 = 0.0739, β = 24.8517, and T_2 = 0.4625. The parameters of controller are listed in Table 3. The transfer function of controller is

$$G_{c1}(s) = \frac{72.9705(0.0739s+1)(0.4625s+1)}{(0.0126s+1)(11.4939s+1)} = \frac{17.281(s+13.54)(s+2.162)}{(s+79.67)(s+0.087)}.$$

The controller is lead-phase (or lag-lead) controller. Moreover, the transfer function of the closed loop is

$$T(s) = \frac{G_c(s)G_p(s)}{1+G_c(s)G_p(s)} = \frac{2419.3(s+13.54)(s+2.162)}{(s+15.45)(s+2.077)(s^2+76.23s+2214)}.$$

The time-domain specifications and deviation ratio are listed in Table 4. In addition, the step response of a compensated system is shown in Figure 4. The deviation ratio of rise time is 0.0302, and the other deviation ratio is zero. Suppose the rise time is very impotent. Then, the weight of DR(T_r) can be increases to 6, while keeping the other weights unchanged. That is

$$w_3 = 6, \text{ and } w_i = 1 \text{ for } i \ne 3.$$

Redesign the controller. When the design procedure is completed, the controller parameters are K= 74.6200, α = 0.0100, T_1 = 99.9949, β = 0.0182, and T_2 = 1.2745. The parameters of controller are listed in Table 3. The transfer function of controller is

$$G_{c2}(s) = \frac{74.6200(99.9949s+1)(0.0182s+1)}{(s+1)(0.0232s+1)} = \frac{410740(s+0.01)(s+0.7846)}{(s+1)(s+43.19)}.$$

The controller is lead-lead (or two-stage phase-lead) controller. Furthermore, the transfer function of the closed loop is

$$T(s) = \frac{5.7503\cdot10^7\cdot(s+0.01)(s+0.7846)}{(s+0.7846)(s+0.01004)(s^2+57.39s+5.75\cdot10^7)}.$$

The time-domain specifications and deviation ratio are listed in Table 4. Besides, the step response of a compensated system is shown in Figure 4. The deviation ratio of rise time DR(T_r) is 0. But DR(T_d) become 0.3, DR(T_s) is 0.08, and DR(T_p) is 0.0769. When designing controller design, it may not be satisfied with all the specifications. In this case, the designer needs to make trade-offs.

Table 2: Example 1 desired value and interval of specifications

Spec.	Desired Value	Interval
T_P	0.1	[0.0990, 0.1010]
T_m	0.1	[0.0990, 0.1010]
T_r	0. 5	[0.4950, 0.5050]
T_d	0.03	[0.0297, 0.0303]
%OS	0.03	[0.0 0.03]
%US	0.02	[0.0, 0.02]
T_s	0.1	[0.0990, 0.1010]
E_{ss}	0.02	[0.0, 0.02]

Table 3: Example 1 parameters of controller

Parameter	$G_{c1}(s)$	$G_{c2}(s)$
K	72.9705	74.6200
α	0.1699	0.0100
T_1	0.0739	99.9949
β	24.8517	0.0182
T_2	0.4625	1.2745

Table 4: Desired interval and deviation ratio in Example 1

Spec.	Interval	G_{c1}	DR_{Gc1}	G_{c2}	DR_{Gc2}
T_P	[0.0990, 0.1010]	0.0992	0	0.1074	0.0769
T_m	[0.0990, 0.1010]	0.0992	0	0.1074	0.0769
T_r	[0.4950, 0.5050]	0.0480	0.0302	0.0495	0
T_d	[0.0297, 0.0303]	0.0303	0	0.0208	0.3000
$\%OS$	[0.0, 0.03]	0.0285	0	0.0300	0
$\%US$	[0.0, 0.02]	0.0200	0	0	0
T_s	[0.0990, 0.1010]	0.1008	0	0.1091	0.0800
E_{ss}	[0.0, 0.02]	0.0039	0	0.0038	0

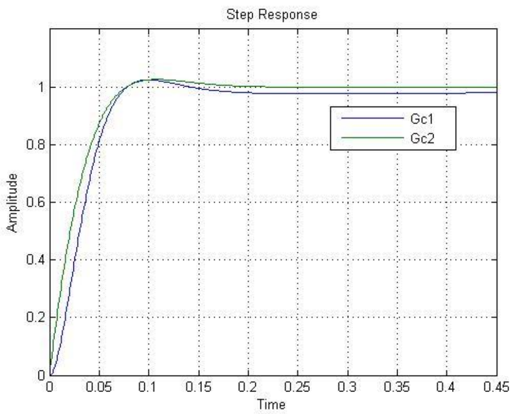

Figure 5: Compensated step response in example 1.

***Example* 2:** Suppose the plant transfer function is given by

$$G_p(s) = \frac{170}{s(s^2 + 17s + 70)}.$$

The step response of a uncompensated system is shown in Figure 5. Here, the peak time is 1.1624 seconds, rise time is 0.5382, maximum overshoot is 8.32%, and setting time is 1.2032 seconds. The steady-state errors for inputs is zero, and a ramp input is 0.4118. Then, let both the first peak time and maximum peak time are 0.7 seconds. In addition, the maximum overshoot is 3%. Following the same method as shown in Example 1, the desired value and interval of specifications are listed in Table 4.

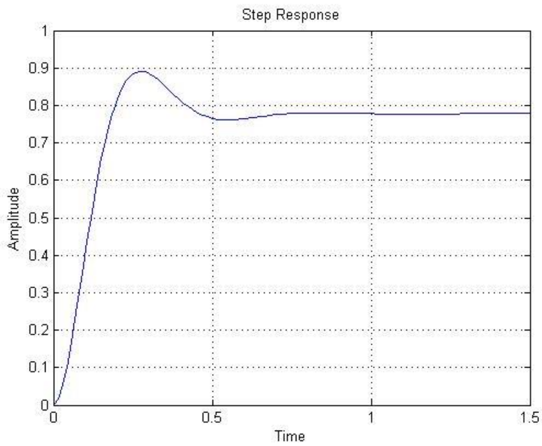

Figure 6: Uncompensated step response in Example 2.

Table 5: Desired Value and Interval of Specifications in Example 2

Spec.	Desired Value	Desired Interval
T_P	0.7	[0.6930, 0.7070]
T_m	0.7	[0.6930, 0.7070]
T_r	0.35	[0.3465, 0.3535]
T_d	0.03	[0.1980, 0.2020]
%OS	0.03	[0.0 0.03]
%US	0.02	[0.0, 0.02]
T_s	0.1	[0.7128, 0.7272]
E_{ss}	0.02	[0.0, 0.2]

The plant in Example 2 is type 1 system. Therefore, the steady state error due to step input is zero. Let error due to ramp input is 0.2. Then, the desired interval of steady state error is determined $0.008 \le E_{ss} \le 0.22$, and the gain K of lead-lag-like controller can be computed using equation 3, i.e. $1.8717 \le K \le 51.4706$. The other parameters of controller, i.e. α, T_1, β and T_2 are set to the interval [0.01, 100].

When the design procedure using MPSO is completed, the controller parameters are K= 48.9458, α= 26.3583, T_1 = 12.8058, β= 0.0100, and T_2 = 0.1715. The controller parameters is listed in Table 7. The transfer function of controller is

$$G_{c1}(s) = \frac{48.9458(12.8058s+1)(0.1715s+1)}{(337.5390s+1)(0.0017s+1)} = \frac{185.68(s+0.07809)(s+5.831)}{(s+0.002963)(s+583)}.$$

The controller is lead-lag (or lag-lead) controller. Furthermore, the transfer function of the closed loop is

$$T(s) = \frac{31566\,(s+5.831)\,(s+0.07809)}{(s+583.1)\,(s+4.816)\,(s+0.07943)\,(s^2+12.01s+64.43)}$$

The time-domain specifications and deviation ratio are listed in Table 6. The step response of a compensated system is shown in Figure 7. Suppose the rise time is critical, which must be matched with the desired interval. Now the weights must be changed. The weight of DR(T_r) is increased to 6, but the other is unchanged. That is

$$w_3 = 6, \text{ and } w_i = 1 \text{ for } i \neq 3.$$

Design the controller once more. When the design procedure is finished, the controller parameters are K= 1.9877, α = 0.0133, T_1 = 0.0265, β = 0.0325, and T_2 = 0.1281. The controller parameters are listed in Table 7. The transfer function of controller is

$$G_{c2}(s) = \frac{1.9877(0.0265s+1)(0.1281s+1)}{(35346s+1)(0.0042s+1)} = \frac{4587.7\,(s+37.7)\,(s+7.807)}{(s+2829)\,(s+240.1)}.$$

The controller is lead-lead (or two-stage phase-lead) controller. Furthermore, the transfer function of the closed loop is

$$T(s) = \frac{779910\cdot(s+37.7)\,(s+7.807)}{(s+2829)\,(s+239)\,(s+8.335)\,(s^2+9.663s+40.72)}.$$

The time-domain specifications and deviation ratio are listed in Table 6. The step response of compensated system is shown in Figure 7. The performance fully meet the desired when the weight of DR(T_r) is increased to 6.

Table 6: Example 2 parameters of controller

Parameter	$G_{c1}(s)$	$G_{c2}(s)$
K	48.9458	1.9877
α	26.3583	0.0133
T_1	12.8058	0.0265
β	0.0100	0.0325
T_2	0.1715	0.1281

Table 7: **Desired interval and deviation ratio in Example 2**

Spec.	Interval	G_{c1}	DR_{Gc1}	G_{c2}	DR_{Gc2}
T_P	[0.6930, 0.7070]	0.7057	0	0.7057	0
T_m	[0.6930, 0.7070]	0.7057	0	0.7057	0
T_r	[0.3465, 0.3535]	0.3220	0.3465	0.3466	0
T_d	[0.1980, 0.2020]	0.2020	0.0001	0.2016	0
%OS	[0.0 0.03]	0.0200	0	0.0296	0
%US	[0.0, 0.02]	0	0	0.0008	0
T_s	[0.7128, 0.7272]	0.7173	0	0.7173	0
E_{ss}	[0.0, 0.2]	0.0084	0	0.2072	0

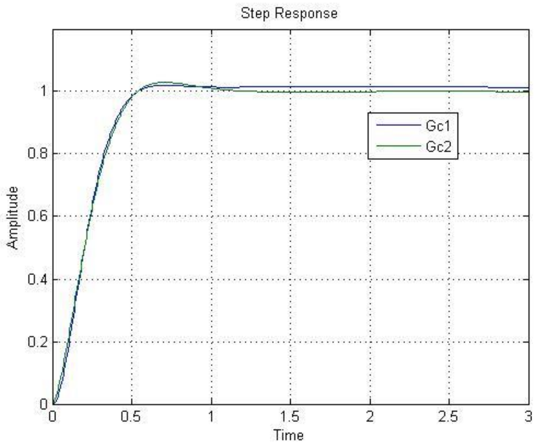

Figure 7: Compensated step response in Example 2.

Example 3**:** A unity feedback system has the following forward transfer function:

$$G_p(s) = \frac{600}{(s+2)(s+4)(s+8)}.$$

The closed-loop system is unstable. The step response of the a uncompensated system is shown in Figure 5. Now, let the first peak time is 1.5 second, and maximum peak time is 3%. Moreover, the maximum overshoot is the same value. Following the same procedure as shown in Example 1, the desired value and interval of specifications are listed in Table 8. Then the MPSO runs 31 times. When the design procedure is finished, the lead-lag parameters are K= 5.8725, α= 50.4539, T_1 = 1.0628, β= 0.0100, and T_2 = 0.1599. The controller parameters are listed in Table 9.

The transfer function of controller is

$$G_{c1}(S) = \frac{5.8725(1.0628s+1)(0.1599s+1)}{(53.6226s+1)(0.0016s+1)} = \frac{11.639\,(s+0.9409)(s+6.252)}{(s+0.01865)(s+625.2)}.$$

The controller is lead-lag (or lag-lead) controller. Furthermore, the transfer function of the closed loop is

$$T(s) = \frac{6983.6\,(s+6.252)\,(s+0.9409)}{(s+625.2)\,(s+7.464)\,(s+0.9121)\,(s^2+4.624s+9.739)}$$

The time-domain specifications and deviation ratio are listed in Table 10. The step response of a compensated system is shown in Figure 9. The deviation ratio of delay time would be improved.

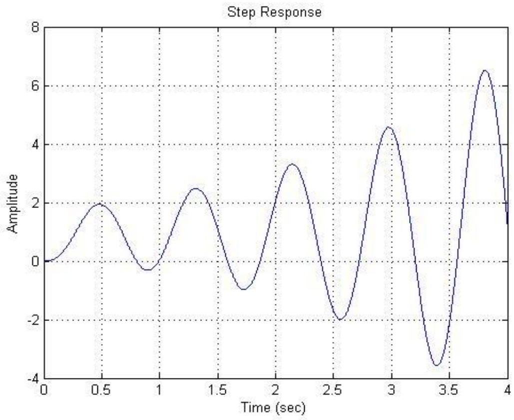

Figure 8: Uncompensated step response in Example 3

Table 8: Desired value and interval of specifications in Example 3

Spec.	Desired Value	Desired Interval
T_P	1.5	[1.4850, 1.5150]
T_m	1.5	[1.4850, 1.5150]
T_r	0.8	[0.7920, 0.8080]
T_d	0.45	[0.4455, 0.4545]
$\%OS$	0.03	[0.0, 0.03]
$\%US$	0.02	[0.0, 0.02]
T_s	1.5	[1.4850, 1.5150]
E_{ss}	0.01	[0.009, 0.011]

We increase the weight of delay time to 6, and keep the others the same. Moreover, the lead-lag parameters are found, K= 6.3105, α= 55.2130, T_1 = 0.4292, β= 0.0010, and T_2 = 0.4308. The controller parameters are listed in Table 9. The deviation ratio of delay time can be improved as shown in Table 10 and Figure 9. Suppose the rise time is critical, and must be matched with the desired interval. Now the weights must be changed. The weight of DR(T_r) is increased to 6, on the other hand the other is unchanged. That is

$$w_3 = 6, \text{ and } w_i = 1 \text{ for } i \neq 3.$$

Design the controller for a second time. When the design procedure is done, the controller parameters are K= 5.3206, α= 0.0100, T_1 = 0.1100, β= 49.8062, and T_2 = 1.1354. The parameters of controller are listed in Table 9. The transfer function of controller is

$$G_{c2}(s) = \frac{5.3206(0.1100s+1)(1.1354s+1)}{(0.0011s+1)(56.5499s+1)} = \frac{10.683\,(s+9.09)\,(s+0.8807)}{(s+909)\,(s+0.01768)}.$$

The controller is lead-lead (or two-stage phase-lead) controller. Besides, the transfer function of the closed loop is

$$T(s)=\frac{6409.6\,(s+9.09)\,(s+0.8807)}{(s+909)\,(s+8.19)\,(s+0.8258)\,(s^2+3.994s+8.43)}.$$

The time-domain specifications and deviation ratio are listed in Table 10. The step response of a compensated system is shown in Figure 9. The deviation ratio of rise time DR(*Tr*) is 0. Nonetheless DR(T_d) becomes 0.1520, DR(T_s) is 0.0555, and DR(T_p) is 0.0391. In this case, the designers must make trade-offs in a variety of specifications..

Table 9: Example 3 parameters of controller

Parameter		$G_{c1}(s)$	$G_{c2}(s)$
K		5.8725	5.3206
α		50.4539	0.0100
T_1		1.0628	0.1100
β		0.0100	49.8062
T_2		0.1599	1.1354

Table 10: Desired interval and deviation ratio in Example 3

Spec.	Interval	G_{c1}	DR_{Gc1}	G_{c2}	DR_{Gc2}
T_P	[1.4850, 1.5150]	1.4875	0	1.5742	0.0391
T_m	[1.4850, 1.5150]	1.4875	0	1.5742	0.0391
T_r	[0.7920, 0.8080]	0.7285	0.0802	0.7920	0
T_d	[0.4455, 0.4545]	0.4545	0	0.5236	0.1520
%OS	[0.0, 0.03]	0.0200	0	0.0200	0
%US	[0.0, 0.02]	0.0044	0	0.0109	0
T_s	[1.4850, 1.5150]	1.5123	0	1.5991	0.0555
E_{ss}	[0.009, 0.011]	0.0090	0	0.0099	0

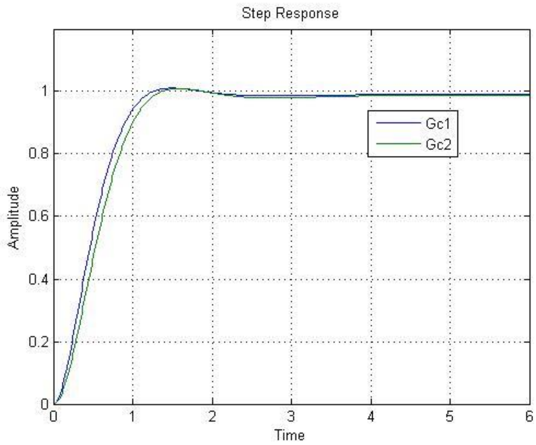

Figure 9: Compensated step response in Example 3

8. Conclusions

In industry, most of the systems can be represented by a linear time-invariant transfer function. This paper focused on the design of lead-lag-like controller. Lead-lag-like controller can be two-stage phase-lead, two-stage phase-lead, lag-lead or lag-lag controller. MPSO overcomes the drawback of PSO, and tries to force all the particles to search the optimum exhaustedly. The proposed objective function includes time-domain specifications, including the delay time, rise time, first peak time, maximum peak time, maximum overshoot, maximum undershoot, setting time and steady state error. Usually designers must make trade-offs in a variety of specifications. As long as the plant could be modeled as a linear time-invariant transfer function, the suggested method would design the lead-lag-like controller capable of approaching the desired specifications. Computer simulations justify the usefulness of the presented method. The result shows the proposed design method obtains better performance. The time-domain specifications can be fully met or very close to the desired.

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