Research on the Detection Method of Inter-Harmonic Based on MUSIC Algorithm

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Abstract

Aiming at the problem of inter-harmonics in power systems, MUSIC algorithm based on subspace decomposition algorithms has been proposed. The signal self-correlation matrix is changed into the signal subspace and the noise subspace on the basis of characteristic value decomposition theory of signal autocorrelation matrices. The noise subspace is further decomposed by orthogonality of two subspaces to get the characteristic polynomial which is constructed based on the decomposition of the noise subspace, and then to solve the polynomial to obtain the signal fundamental frequency and harmonic frequency estimation. Finally, the signal amplitude and phase are detected by the least squares method. Compared with other classical algorithms, the results show that the proposed algorithm is feasible, efficient and stable.

Keywords: electrical power system; harmonic analysis; inter harmonic analysis; MUSIC algorithm

1. Introduction

With the increasing development of power electronic technology, a large number of non-linear load fluctuation, various frequency conversion speed regulating device and power electronic devices are widely used in power systems, causing the power system voltage and current waveform distortion, and resulting in more serious harmonic in the power system . At the same time, the non integer harmonics – inter-harmonics and sub-harmonics, also have caused the widespread concern[1] in scholars at home and abroad. The presence of inter-harmonics in the electrical power system leads to the current distortion and the current fluctuation of users' side, so it is needed to analyze the inter-harmonics spectrum accurately at the same time[2].

At present, the common analysis methods are FFT algorithm[3], wavelet algorithm[4, 5], instantaneous reactive power theory[6], multi signal classification method (MUSIC)[7, 8], etc.. There are many advantages and disadvantages in every method. Especially, the noise ratio detection accuracy in low signal is greatly affected.

In this paper, the signal self-correlation matrix is changed into the signal subspace and the noise subspace on the basis of characteristic value decomposition theory of signal autocorrelation matrices. The noise subspace is further decomposed by orthogonality of two subspaces to get the characteristic polynomial which is constructed based on the decomposition of the noise subspace, and then solve the polynomial to obtain the signal fundamental frequency and harmonic frequency estimation. Finally, the signal amplitude and phase are detected by the least squares method.

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Compared with other methods, the simulation results show that the mentioned method has high frequency resolution, and it has the ability to resist noises without requirements on the data length.

2. Mathematical Model

The voltage (or current) signal of an electrical power system is

$$x(t)=\sum_{i=1}^{N}A_i\sin(\omega_i t+\varphi_i)+u(t) \tag{1}$$

where $x(t)$ means a voltage or current signal; N means the number of inter-harmonics; $u(t)$ means white noise; A_i means the amplitude of i harmonic; $\omega=2\pi f$ means the frequency of the i inter-harmonics; φ_i means the initial phase of the i inter-harmonic.

Characteristic polynomial is calculated based on complex signals, and the original signal can be transformed into a spatial signal by Euler formula. Equation (1) can be changed into a complex frequency expression:

$$x(t)=\sum_{i=1}^{p}A_i\frac{e^{j(\omega_i t+\varphi_i)}-e^{-j(\omega_i t+\varphi_i)}}{2j}+v(t) \tag{2}$$

3. Theoretical Basis of Spatial Spectrum Estimation

In this paper, we assume that every element of noise vectors is White Gaussian Noise with the zero mean value. They are independent of each other and have the same variance σ^2. According to the definition of self-correlation matrix:

$$\begin{aligned}R&=E[\mathbf{x}(n)\mathbf{x}^H(n)]\\&=A(\omega)E[\mathbf{s}(n)\mathbf{s}^H(n)]A^H(\omega)+\sigma^2 I\\&=AR_sA^H+\sigma^2 I\end{aligned} \tag{3}$$

and the theory of characteristic value decomposition (detailed theory can be found in the literature [9]), the self-correlation matrix can be decomposed into:

$$\begin{aligned}U^HRU&=U^HAR_sA^HU+\sigma^2U^HU\\&=diag(\alpha_1^2,\alpha_2^2,\cdots,\alpha_p^2,0,\cdots,0)+\sigma^2 I\end{aligned} \tag{4}$$

The characteristic value of the self-correlation matrix is:

$$\lambda_i=\sigma_i^2=\begin{cases}\alpha_i^2+\sigma^2, i=1,2,\cdots,p\\ \sigma^2, i=p+1,p+2,\cdots m\end{cases} \tag{5}$$

This shows that when the AWGN is observed, it is easy to distinguish the fist 2P characteristic value between the last $M-2p$ ones of self-correlation matrix R. Therefore, the first 2p characteristic values are the main signal characteristic values, and the rest $M-2p$ characteristic values are the noise characteristic values. According to the signal characteristic values and the noise characteristic values, the column vector of the characteristic matrix can be divided into 2 parts, which are composed of the signal feature vector and the noise characteristic vector.

$$U=\begin{bmatrix}S & G\end{bmatrix} \tag{6}$$

4. MUSIC Algorithm Principle

The multiple signal classification is the method to sort the characteristic value of the signal self-correlation matrix R. Then the feature vector is constructed, and finally the parameters of the spatial signal are estimated by the noise subspace feature vector matrix.

From the Equation (6), we can know that the signal subspace $S = span\{s_1, s_2, \cdots, s_p\}$ and noise subspace $G = span\{g_1, g_2, \cdots, g_{m-p}\}$ are composed of different feature vectors. Then the matrix R in formula (6) can be expressed as:

$$R = S\Sigma_S S^H + G\Sigma_G G^H \tag{7}$$

In Equation (7), $\Sigma_S = diag(\lambda_1, \lambda_2, \cdots, \lambda_p)$ and $\Sigma_G = diag(\sigma, \sigma, \cdots, \sigma)$. We can know that the signal subspace and the noise subspace are mutually orthogonal, we get: $\mathbf{a}^H(\omega_i)G = \mathbf{0}(i = 1,2,\cdots, p)$.

In the actual sampling signal, the signal data matrices collected by the equal interval linear array are finite, so the maximum likelihood estimation is needed for the self-correlation matrix R:

$$\hat{R} = \frac{1}{N}\sum_{n=1}^{N}\mathbf{x}(n)\mathbf{x}^H(n) \tag{8}$$

We can decompose characteristic value for $\hat{R}$ in the Equation (8), and then get the noise subspace feature vector matrix $\hat{G}$.

Due to the noise disturbance in the signal sampling, results in the Equation (3.38) cannot be fully established, and the practical application of MUSIC spectrum estimation for direction of arrival is to optimize the minimum search implementation, we get:

$$\omega = \arg\min \mathbf{a}^H(\omega)\hat{G}\hat{G}^H\mathbf{a}(\omega) \tag{9}$$

Then the spatial spectrum estimation formula of the MUSIC algorithm can be defined as:

$$P_{MUSIC} = \frac{1}{\mathbf{a}^H(\omega)\hat{G}\hat{G}^H\mathbf{a}(\omega)} = \frac{1}{\sum_{i=p+1}^{m}\left|\mathbf{a}^H(\omega)\hat{G}\right|^2} \tag{10}$$

In practical applications, at every frequency of each harmonic or inter-harmonic ω_i in the signal, $P_{MUSIC}(\omega)$ will be at a peak. Therefore, ω usually is divided for hundreds of equal space units, and every small unit $\Delta\omega$ needs step size. Searching the P peak of $P_{MUSIC}(\omega)$ in $[0,2\pi]$ means every ω_i is put into Equation (10). The corresponding ω of each peak value is calculated by the peak search method, and the normalized frequency is calculated by:

$$f_i = \frac{\omega_i}{2\pi} \tag{11}$$

The computation of MUSIC algorithm is mainly derived from the following aspects: Firstly, we need to obtain all the characteristic values and feature vectors of the matrix R. Secondly, we need to use each of the noise characteristic vectors and the direction vectors $\mathbf{a}(\omega_i)$ of the signal to multiply operations, and the space spectrum function $P_{MUSIC}(\omega)$ of the MUSIC algorithm is obtained by the sum of the Euclidean norm. Thirdly, the frequency spectrum peak search of the whole frequency domain ω_i on the spatial spectrum function $P_{MUSIC}(\omega)$ is needed.

Based on all the analysis above, the operation steps of the MUSIC algorithm are given as follows:

1) According to the Equation (7), the signal data covariance matrix R is obtained from the array data which is composed of the harmonic signal sampling;
2) The characteristic value decomposition of signal data covariance matrix R is calculated according to the equation $R = U\Sigma U^H$;
3) Judge the number of signal sources according to the characteristic values decomposed from R, and determine the main characteristic values $\lambda_1, \lambda_2, \cdots, \lambda_p$ corresponding to the signal subspace and the sub characteristic values σ^2 corresponding to the noise subspace;
4) The signal subspace $S = span\{s_1, s_2, \cdots, s_p\}$ and the noise subspace $G = span\{g_1, g_2, \cdots, g_{m-p}\}$ are span by the classified characteristic values according the previous step;
5) After calculating the spectrum in the signal range according to the Equation (10), we can get the spectral function of the MUSIC algorithm $P(\omega_i)$, and $\omega_i = (i-1)\Delta\omega$, and then make full spectrum peak search;
6) Finding out the p peak of the $P(\omega)$, they are the estimated value $\omega_1, \omega_2, \cdots, \omega_p$ of MUSIC that we want to know.

5. Inter-Harmonic Frequency Detection Process

The amplitude of the inter-harmonic component of the power system signal is less than a few percent amplitude of the fundamental wave component or even smaller. When we make the non synchronous sampling towards inter-harmonic component, the frequency component of the small amplitude [3] is likely to be drowned out by the spectrum leakage of the large value frequency component.

In this paper, the decreasing processing ideas mentioned in the literature[10] is regarded as the method for detecting process. The main idea is: large signal in the original signal is detected firstly, in order to eliminate the impact of large signal spectrum leakage on small signal, the original signal minus the large signal in order to detect small signal. In this way, we will not lose the original signal information, and can improve the accuracy of detection.

The signal for the power system is showed by $x(t)$, so:

$$\begin{aligned} x(t) &= v_h(t) + v_i(t) \\ &= v_i(t) + A\sin(2\pi ft + \varphi) \\ &= v_i(t) + a\sin(2\pi ft) + b\cos(2\pi ft) \end{aligned} \tag{12}$$

In the last equation, $v_h(t)$ means large amplitude signal; $v_i(t)$ means small amplitude signal; and $a = A\cos\varphi, b = A\sin\varphi$.

We make sampling towards power system signals on the basis of Nyquist sampling theorem, and change the Equation (12) to vector form:

$$\begin{aligned} X &= RW + e \\ X &= [x(t_1), x(t_2), \cdots, x(t_N)]_{N\times 1} \\ R &= \begin{bmatrix} \sin(2\pi ft_1) & \cos(2\pi ft_1) \\ \sin(2\pi ft_2) & \cos(2\pi ft_2) \\ \vdots & \vdots \\ \sin(2\pi ft_N) & \cos(2\pi ft_N) \end{bmatrix}_{N\times 2} \end{aligned} \tag{13}$$

In the last equation, $W = [a \quad b]^T$ means estimated coefficient; e means error vector; N means sampling coefficient.

$W = [a \quad b]^T$ is calculated for pre-estimate processing by least square method, we get:

$$W = \left[R^T R\right]^{-1} R^T X \tag{14}$$

Because the frequency of the Equation (11) has been determined by the MUSIC algorithm, the matrix R is known, and the vector X is the sampling signal; therefore, W can be easily obtained according to the Equation (14), and $v_h(t)$ can be determined. Then by the Equation (12) we can know:

$$v_i(t) = x(t) - v_h(t) \tag{15}$$

The spectrum leakage caused by a large amplitude signal have weakened or eliminated because of the loss of substantial amplitude signal in the time domain. Then we will continue to adopt MUSIC algorithm to make spectral analysis towards the signal defined by the Equation (15), and the frequency of small amplitude signals can be detected accurately.

To detect the smaller amplitude of the inter-harmonic component, the large amplitude fundamental and harmonic and the large amplitude inter-harmonics in the power system signal are lost in order by this method. It can effectively restrain the interference of the fundamental, harmonic and inter-harmonic, and further improve the precision and frequency resolution of frequency detections.

6. Inter-harmonic Amplitude and Phase Detection

We can get the frequency and number of the harmonic through the above process, and the following processes aim at obtaining the signal amplitude and phase by a least square method. It is assumed that N is the number of signal harmonics and inter-harmonics estimated by MUSIC algorithm s , and ω_i is the frequency. For the harmonic time function without a direct component:

$$x(t) = \sum_{i=1}^{N} A_i \sin(\omega_i t)\cos\varphi_i + \sum_{i=1}^{N} A_i \cos(\omega_i t)\sin\varphi_i \tag{16}$$

Each sampling value of the signal should be satisfied with the Equation (16). If we know the p sampling values of $x(t)$ regarded as $x(t_1), x(t_2), \cdots, x(t_p)$, we can get p equations changed into the matrix form:

$$\begin{bmatrix} \sin\omega_1 t_1 & \cos\omega_1 t_1 & \cdots & \sin\omega_N t_1 & \cos\omega_p t_1 \\ \sin\omega_1 t_2 & \cos\omega_1 t_2 & \cdots & \sin\omega_N t_2 & \cos\omega_N t_2 \\ \vdots & \vdots & & \vdots & \vdots \\ \sin\omega_1 t_p & \cos\omega_1 t_p & \cdots & \sin\omega_N t_p & \cos\omega_N t_p \end{bmatrix} \times \begin{bmatrix} A_1\cos\varphi_1 \\ A_1\sin\varphi_1 \\ \vdots \\ A_N\cos\varphi_N \\ A_N\sin\varphi_N \end{bmatrix} = \begin{bmatrix} x(t_1) \\ x(t_2) \\ \vdots \\ x(t_p) \end{bmatrix}$$

The simplified matrix is expressed as:

$$\underset{p\times 2N}{[A]}\,\underset{2N\times 1}{[X]} = \underset{p\times 1}{[B]} \tag{17}$$

According to Equation (17), 2N is the number of variables in the equations, and $p > 2N$ is often established in practical applications. The size of matrix $[B]$ must be increased to improve the detection accuracy. If the Equation (17) is left by the Pseudo inverse matrix $[A]^+$ of the matrix $[A]$ on both sides of the equation, the expression of the matrix $[X]$ can be obtained:

$$\underset{2N\times 1}{[X]} = \underset{2N\times p}{[A]^+}\,\underset{p\times 1}{[B]} \tag{18}$$

All the elements in the matrix $[X]$ including the amplitude and phase parameters can be solved by the Equation (18), and the amplitude and phase parameters of the harmonic and inter-harmonic in the original signal are obtained by the following equation:

$$\begin{cases} A_i = \sqrt{(A_i \cos\varphi_i)^2 + (A_i \sin\varphi_i)^2} \\ \varphi_i = arc\cot \dfrac{A_i \cos\varphi_i}{A_i \sin\varphi_i} \end{cases} \tag{19}$$

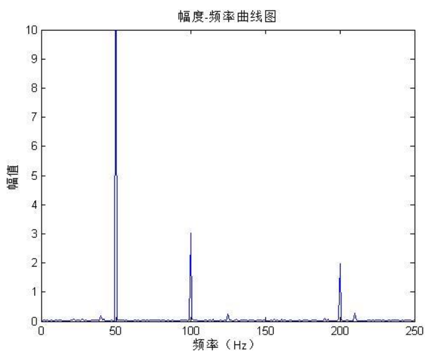
7. Simulation Examples and Performance Analysis

According to the characteristics of the power system signal (without noise), the signal is set to be detected:

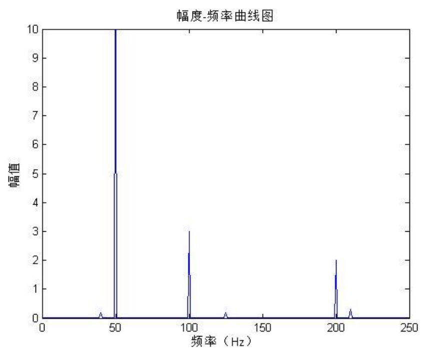
$$x(t) = 0.2\sin(2\pi \times 40t + 60^\circ) + 10\sin(2\pi \times 50t) + 3\sin(2\pi \times 100t + 40^\circ) + 0.2\sin(2\pi \times 125t + 70^\circ) + 2\sin(2\pi \times 200t + 25^\circ) + 0.3\sin(2\pi \times 210t + 80^\circ)$$

The signal fundamental frequencies are 50Hz, 100Hz and 200Hz harmonics. 40Hz, 125Hz and 201Hz of the harmonic components are set respectively according to the inter harmonic characteristics; the sampling frequency is 1000Hz; the number of array elements is 40; the sampling point is 1000.

In order to verify the performance of this algorithm, we compare the simulation result of MUSIC algorithm with the result of FFT algorithm by MATLAB software in the case of SNR of 10dB and 30dB. The results are shown in figure 1.



(a)

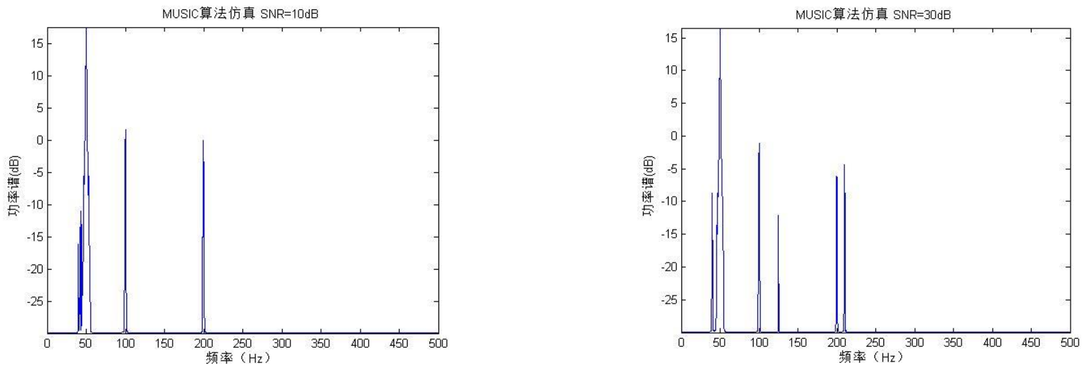


(b)

Figure 1: The spectrum estimation of the interpolation FFT algorithm. (a) SNR of 10dB (b) SNR of 30dB

When the SNR is 10 dB, FFT algorithm cannot detect small amplitudes of inter-harmonic components as shown in Figure 1 (a); only when the SNR reaches 30dB, the inter-harmonic frequency can be detected as shown in Figure 1 (b). And there is a big error in this method. It shows that the FFT algorithm is sensitive to noises, and its frequency resolution is f_s/N; its frequency resolution is also relatively low 3 in the case of less data.

Figure 2 shows the MUSIC method of power spectrum estimation. As can be seen, MUSIC algorithm improves the frequency resolution in a certain extent, and this method has certain anti-noise ability; however, fundamental frequency spectrum leakage is more serious in low SNR. The inter harmonics of the 40 Hz are completely submerged by the noise of the fundamental frequency spectrum, and the pseudo spectrum of the 210Hz is not detected, the frequency of power system signal may have some error determination. The results are shown in figure 2 (a).



(a) (b)

Figure 2: Spectral estimation of MUSIC algorithm. (a) SNR of 10dB (b) SNR of 30dB

Table 1: Parameter estimation of power system signal under different signal to noise ratio

Original signal parameter setting			SNR	Detection results of FFT algorithm			Detection results of MUSIC algorithm		
Frequency /Hz	Amplitude /V	Phase		Frequency /Hz	Amplitude /V	Phase	Frequency /Hz	Amplitude /V	Phase
40	0.2	60	10dB	—	—	—	—	—	—
			30dB	39.9608	0.2305	57·8325	40.0639	0.2203	60/1257
50	10	0	10dB	49.9500	9.9593	0.2569	50.0102	9.8192	0.1297
			30dB	49.9501	9.9633	0.1298	50.0025	9.9827	0.0013
100	3	40	10dB	99.9901	2.9782	38.6954	99.9880	2.8192	40.2732
			30dB	99.9301	2.9721	39.2357	100.0037	3.0137	39.8927
125	0.2	70	10dB	—	—	—	124.9123	0.3536	70.5809
			30dB	124.875	0.1968	68.9378	125.0465	0.2147	70.1653
200	2	25	10dB	199.800	1.8066	30.2651	200.0319	1.8192	24.8967
			30dB	199.800	1.8168	24.3798	200.0045	1.9968	25.1028
210	0.3	80	10dB	—	—	—	209.8535	0.3653	81.4870
			30dB	209.790	0.2227	82.1987	209.9876	0.3120	79.8923

With the improvement of signal to noise ratio, when the SNR reaches 30dB, the harmonic components of 40Hz and 210Hz can be detected as shown in Figure 2 (b), and the frequency detection accuracy is better than FFT.

Table 1 shows that the FFT algorithm and the MUSIC method cannot accurately detect the frequency components of the power system signal in low SNR. FFT algorithm and MUSIC algorithm are likely to draw small amplitude of the inter-harmonic component in the case of low SNR. When the SNR becomes high enough, FFT algorithm can detect inter-harmonics components; however, the frequency estimation accuracy is low. While the MUSIC algorithm can accurately distinguish harmonic components from inter-harmonic components, and it overcomes the lack of the harmonic component to be detected in the low signal to noise ratio. At the same time, when the SNR is the same, the accuracy of the MUSIC algorithm is much higher than that of the FFT algorithm. The phase and amplitude parameters of a power system can be estimated accurately by extracting the simulation data and the least square method when the frequency reaches a high precision.

8. Conclusion

In this paper, based on the study of spatial spectrum estimation, the MUSIC algorithm is constructed by analyzing the characteristics of the noise space. It is applied to the signal detection in the power system to get the estimate of the frequency components of the signals in the power system, detect the amplitude and phase of the signals in the power system by the least square method, and achieve good spectrum effects, especially in the detection of inter-harmonic components. This method overcomes the shortcomings of FFT algorithm in the case of short data, and the method has a higher accuracy than the FFT algorithm. There are some errors in estimation of phase and amplitude of power system signals calculated by least square method because the least square method is more sensitive to noises. Then this method needs to be further improved. All in all, the new algorithm proposed in this paper has good stability, high resolution and wide application ranges. It has a certain significance for real-time accurate detection of power system signals.

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