

Lifetime Prediction of a Lithium-ion Battery Based on a Phase Space Reconstruction Support Vector Machine

Zhi-feng Liu, * Sheng-hua Chen, Li-peng Cao and Guo-qian Lin

Abstract

With the increasing number of use times, not only the performance of the lithium-ion battery decreases, but also the potential safety hazard increases. Through the prediction of the lithium-ion battery life, the lithium-ion battery can be controlled effectively according to the predicted results, so as to reduce the accident rate and improve the reliability of the lithium-ion battery in the application process.

The method of a combining phase space reconstruction and support vector machine is used to predict the remaining life of lithium-ion battery. The noise of original test data was reduced by using wavelets. A part of the test data from the lithium-ion battery capacity is used to train and test the model. Here, the training and testing data for the 1060 groups of data have been measured in the first 700 groups, which accounted for 70% and 30%, respectively. The capacity of the lithium-ion battery was predicted by using the prediction model and the remaining 360 sets of data. The method of a phase space reconstruction support vector machine is used to predict the remaining life of lithium-ion battery. The relative error between the predicted value and the true value is about $\pm 3\%$, and not more than $\pm 5\%$. The result shows that the phase space reconstruction support vector machine model is suitable for the prediction of the remaining life of lithium-ion battery.
Keywords: Wavelet, Support Vector Machine, Phase Space Reconstruction, Lithium-ion Battery, Life Prediction

1. Introduction

Because of its low cost, long service life, and high energy density, a lithium-ion battery has been more and more widely used [1-2]. With the increase of use frequency, the performance of lithium-ion battery will decline, the battery failure leads to the recession of the battery, and the failure of the battery will cause great security risks, so the correct remaining life prediction of the lithium-ion battery is of great significance [3-5].

There are many methods to predict the life of a lithium-ion battery, such as the support vector machine algorithm, the neural network algorithm, the Grey Theory and other algorithms [6-7]. A method based on BP neural network was proposed to predict the remaining life of lithium-ion battery in reference [8], but the training time of BP neural network was slow; the final prediction accuracy was about 10%, so the prediction accuracy of this method was not high. Based on BP neural network optimizing by MIV algorithm, a method was proposed to predict the remaining life of lithium-ion battery in reference [9]. The prediction accuracy was about $\pm 5\%$. This accuracy was greatly improved by using the MIV algorithm to optimize the BP neural network, but it did not solve the problem of long convergence speed of BP neural network training. The method based on LS-SVM was proposed to predict the life of lithium-ion battery in reference [10]. This method used genetic annealing algorithm to optimize the parameters, and then used the LS-SVM method to predict the life of a lithium-ion battery. This method has higher precision, but the algorithm is easy to fall into local optimal solution and the convergence rate is slow. The method based on QPSO-SVM was proposed to predict the life of lithium-ion batteries in reference [11]. This method used the quantum behaved particle swarm algorithm to optimize the SVM parameters, but the method did not solve the problem of a single input. A reliable method is proposed to predict the lifetime of lithium-ion battery, which is based on the method of a phase space reconstruction support vector machine, which is suitable for solving the problem of small samples, high dimensions and nonlinear regressions. Firstly, a wavelet is used to reduce the noise of the original test data to improve the prediction accuracy. A part of the test data of the lithium-ion battery capacity is then taken out to train and test the model. Here, the training and testing data for the 1060 groups of data have been measured in the first 700 groups, which accounted for 70% and 30%, respectively. The capacity of the lithium-ion battery is predicted by using the prediction model and the remaining 360 sets of data. Finally, the relative error of the predicted value and the real value are drawn, and the prediction accuracy of the phase space reconstruction support vector machine is obtained.

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2. Phase Space Reconstruction Support Vector Machine

2.1 Phase Space Reconstruction

The phase space reconstruction technique was first proposed by Packard and others. The technique of phase space reconstruction was to extend the value of one dimensional time series to the high dimensional phase space. The basic idea of phase space reconstruction is that each point of the high dimension phase space is not isolated but connected with each other [12-13]. For a given time series $\{x(n)\}_{n=1}^{\infty}$, if the appropriate time delay parameter τ and the embedding dimension d are selected, the phase space reconstruction state vector is as follows.

$$X(t_i)=[x(t_i),x(t_i+\tau),x(t_i+2\tau),\dots,x(t_i+(m-1)\tau)] \quad (1)$$

If phase space dimension d is too small, phase space dimension can not reflect the real relationship within the data, and the phase space will overlap. Conversely, if the d is too large, the d will cause the gap between the data is too large, resulting in unnecessary noise, and the d cannot correctly reflect the true relationship within the data. The proper embedding dimension will shorten the calculation time, reduce calculation error, and shorten the prediction time. For large samples, the time delay parameter will not have a greater impact on the prediction. However, for small samples, the time delay parameter will have a greater impact on the prediction. If the time delay parameter τ is too large, a power signal distortion is produced by time series description, which will make the problem complicated. If the time delay parameter τ is too small, it will cause the correlation too strong, which will cause the hidden part of data information [14]. Therefore, the key of phase space reconstruction is to select the appropriate phase space dimension d and time delay parameter τ . In the real prediction process, the root mean square error (RMSE) is used to evaluate if the phase space dimension d and the time delay parameter τ are appropriate. The calculation formula is as follows.

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (z_i - z_i^-)^2} \quad (2)$$

2.2 Support Vector Machine (SVM)

Support vector machine is developed on the basis of a statistical theory, which is mainly used to solve the regression problem of small samples, nonlinear and high dimensional spaces. The SVM uses a kernel function to map the input data to a high dimensional space, and solve nonlinear classification and regression problems in a high dimensional space [15-17]. Choosing the nonlinear kernel function and constructing the optimal nonlinear hyper plane are the key to the construction of SVM.

2.2.1 Kernel Function Classification

The kernel function of SVM must satisfy the Mercer theorem. The common kernel functions are linear kernel function, polynomial kernel function, RBF kernel function and S kernel function, among which the RBF kernel function is the most widely used. The RBF kernel function can be applied to large samples, small samples, high and low dimensions. The calculation formula of the RBF kernel function is as follows.

$$K(x_i, x_j) = \exp \left\{ -\frac{|x_i - x_j|^2}{\sigma^2} \right\} \quad (1)$$

where σ is called the kernel width.

2.2.2 Optimal Classification Hyper Plane

When the input sample is two-dimensional and linear, a straight line can be used to classify the input samples. When the input sample is not linear, an appropriate kernel function is selected; the input samples are mapped to a high dimension, and the original classification straight line is turned into a hyper plane. When the data is linear, the classification line is shown in Figure 1.

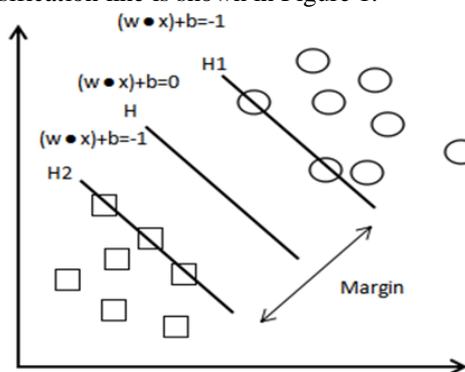


Figure 1: The optimal linear classification

As shown in Figure 1, round and square represent a class of data, respectively, H is the best classification line. H1 and H2 are parallel to H1, and the distance between H1 and H2 is called the classification interval. The optimal classification line can not only separate the sample data, but also require the maximum classification interval between H1 and H2 [18-20]. The linear classification function is as follows.

$$f(x) = w \bullet x + b \tag{4}$$

The classification line is as follows.

$$w \bullet x + b = 0 \tag{5}$$

The classification distance between H1 and H2 is $\frac{2}{\|w\|}$, and the problem of finding maximal value of the classification interval is converted to find the minimum value of $\frac{\|w\|^2}{2}$.

3. Noise Reduction Processing of the Original Test Data of Lithium-ion Battery

3.1 The test Data of a Lithium-ion Battery

The lithium-ion battery with the positive electrode material which composes of nickel, cobalt and manganese materials is used as the experimental object. The NMC battery has the characteristics of large capacity and high discharge rate. By accelerating the aging of NMC battery, the service life of the battery is shortened, and the original test data is recorded in this experiment. When the battery capacity is reduced to 80% of the rated capacity, the battery becomes invalid. The rated capacity of NMC battery is 2.0Ah, and the failure capacity is 1.6Ah. The constant current discharge is used in the experiment. The capacity attenuation of the original test data of the 1060 groups of NMC battery is shown in Figure 2.

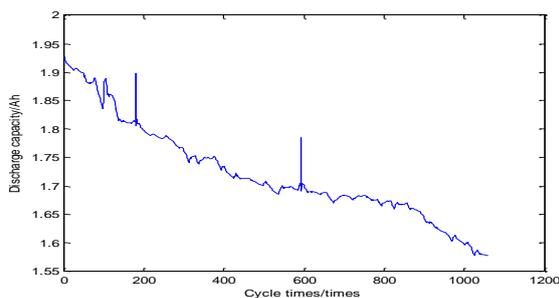


Figure 2: The capacity attenuation of the original test data for Lithium-ion battery

As shown in Figure 2, the real life of NMC battery is 1000 cycles. When the cycle time is more than 1000 times, the NMC battery becomes invalid. The capacity of NMC battery appears mutations in 150 or about 560 times, which demonstrates that the influence of temperature change on NMC battery is great.

3.2 The Noise Reduction Processing of the Original Test Data

The data obtained during the experiment is the key of the whole experiment. If the data error is too much, it will affect the whole prediction result. In the course of the experiment, there are many factors that affect the original test data, such as the temperature difference, the change of discharge rate, the difference of the battery itself and so on. If these data are directly used to train and predict, it will directly affect the accuracy of prediction, and affect the data mining of the whole model. Therefore, before the training and prediction of the data, we need to reduce the noise signal of the original data and improve the training and prediction accuracy of the data.

The wavelet denoising is one of the important research objects of the wavelet theory. The wavelet theory is also widely used in the identification and detection of signals, speech recognition, sample estimation, etc. [21].

If an original signal is $f(n)$, the original signal contains noise signal, $s(n)$, and then the noise model can be expressed as follows:

$$s(n) = f(n) + \sigma e(n) \tag{7}$$

where σ is the noise intensity, and $e(n)$ is the noise signal.

The test data denoising process steps based on the wavelet theory are as follows:

- 1) The 1060 sets of original test data (including noise signal) are decomposed.
- 2) The original test data is decomposed by selecting a suitable wavelet basis function.
- 3) The appropriate threshold is chosen to quantify the wavelet decomposition coefficient, and the general high frequency signal is considered as the noise signal.
- 4) The signal is generated after denoising (wavelet reconstruction).

The flow chart of wavelet denoising process is shown in Figure 3.

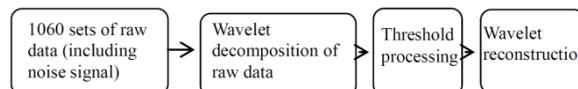


Figure 3: The flow chart of wavelet denoising

The result of denoising is directly affected by the selection of a wavelet basis function, layer number n and the selection of threshold function.

3.2.1 The selection of wavelet basis functions

Commonly used wavelet basis functions are Haar wavelet, DbN wavelet, Sym.N wavelet, Hat Mexican (mexh) wavelet, Morlet wavelet, Meyer wavelet and so on[22-23]. The most commonly used wavelet bases are DbN wavelet and Sym.N wavelet.

3.2.2 The selection of threshold functions

When the threshold function is determined, we can use the hard or soft value function to deal with the threshold value. The hard function formula is as follows:

$$S = \begin{cases} x & |x| > T \\ 0 & |x| \leq T \end{cases} \quad (7)$$

where S is the wavelet transform coefficient after denoising, |x| is the wavelet transform coefficient before denoising, and T is the critical threshold.

In the hard value function, when the wavelet coefficient |x| is less than or equal to the threshold value T, the wavelet coefficient S is 0. When the wavelet coefficient |x| is larger than the threshold value T, the wavelet coefficient S is equal to the absolute value of the wavelet coefficient |x| before denoising.

The soft value function formula is as follows:

$$S = \begin{cases} sign(x)(|x|-T) & |x| > T \\ 0 & |x| \leq T \end{cases} \quad (8)$$

In the soft value function, when the wavelet coefficient |x| is less than or equal to the threshold value T, the wavelet coefficient S is 0. When the wavelet coefficient |x| is larger than the threshold T, the wavelet coefficient S is equal to $sign(x)(|x|-T)$.

3.2.3 The wavelet denoising evaluation index

The wavelet denoising is usually evaluated by the signal-to-noise ratio SNR and root mean square error RMSE [24].

The root mean square error RMSE calculation is as follows:

$$RMSE = \sqrt{\frac{1}{n} \sum_n (f(x_i) - \bar{f(x_i)})^2} \quad (9)$$

The signal-to-noise ratio SNR calculation is as follows:

$$SNR = 10 * \lg(p_s / p_0) \quad (10)$$

where $p_s = \sum_n f(x_i)^2$ $p_0 = RMSE^2$.

In the formula, the root mean square error is bigger, and the signal-to-noise ratio is smaller. The wavelet denoising evaluation standard is that RMSE is smaller, and denoising effect is better [25].

The NMC battery data is denoised by wavelets, the wavelet basis function uses Haar wavelet, and the original test data is decomposed into three layers; and the threshold estimation uses Heuristic. The root mean square error and signal-to-noise ratio of the data after denoising are RMSE=0.0024, SNR=57.2051, respectively. The relative error between the data after denoising and the original test data is shown in Figure 4.

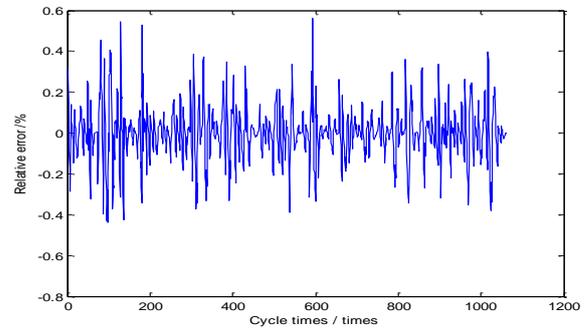


Figure 4: The relative error between the data after denoising and the original test data based on the Haar wavelet

The basis function uses Sym.5 wavelet, and the original test data is decomposed into three layers; and the threshold estimation uses Heuristic. The root mean square error and signal-to-noise ratio of the data after denoising are RMSE=0.0011, SNR=63.7723, respectively. The relative error between the data after denoising and the original test data is shown in Figure 5.

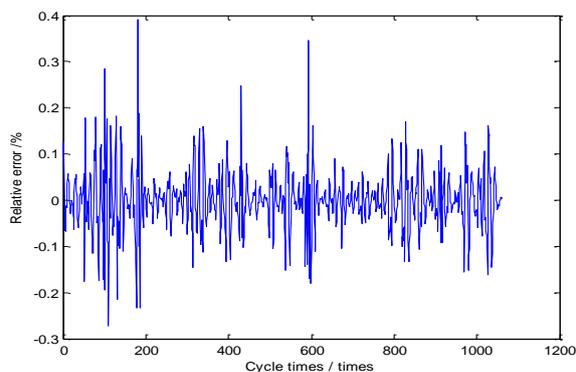


Figure 5: The relative error between the data after denoising and the original test data based on the Sym.5 wavelet

The basis function uses Db5 wavelet, and the original test data is decomposed into three layers; and the threshold estimation uses Heuristic. The root mean square error and signal-to-noise ratio of the data after denoising are RMSE=0.0017, SNR=60.2275, respectively. The relative error between the data after denoising and the original test data is shown in Figure 6.

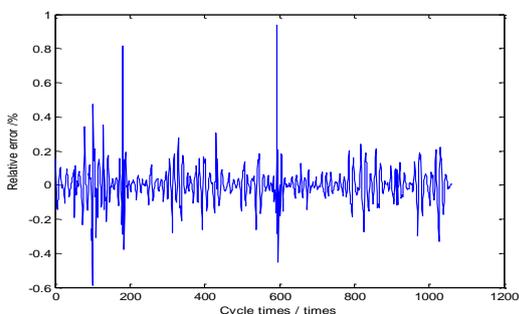


Figure 6: The relative error between the data after denoising and the original test data based on the Db5 wavelet

Table 1: The RMSE and SNR values of the denoising data based on three kinds of wavelet basis functions

Wavelet basis function	Haar wavelet	Sym.5 wavelet	Db5 wavelet
RMSE	0.0024	0.0011	0.0017
SNR	57.2051	63.7723	60.2275

The different wavelet basis functions are used to get the different values of RMSE and SNR after denoising, as shown in Table 1.

As shown in table 1, the Harr wavelet gets RMSE=0.0024 and SNR=57.2051. The Sym.5 wavelet gets RMSE=0.0011 and SNR=63.7723; The Db5 wavelet gets RMSE=0.0017 and SNR=60.2275. Due to the smaller RMSE or the greater SNR, the noise reduction effect is better. By contrast, the original test data is processed by the wavelet basis

function Sym.5. The original test data was denoised by Sym.5 wavelet. The capacity fading of a Lithium-ion battery is shown in Figure 7. The average relative error between the data after denoising and the original test data is shown in Table 2.

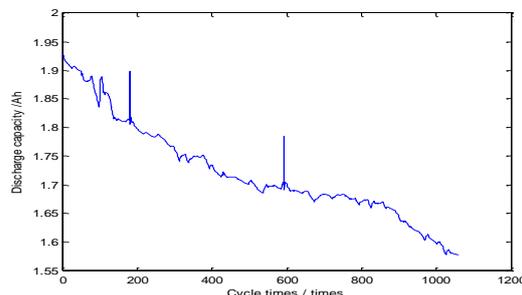


Figure 7: The capacity attenuation of lithium-ion battery after denoising

Table 2 The average relative error between the data after denoising and the original test data

Cycle times / times	1-200	201-400	401-600	601-800	801-1000
Average relative error / %	3.5951	3.4685	3.4008	3.3413	3.1962

As shown in figure 7, the outburst points in the cycle times of 150 times and 560 times still exist, which shows that the test data after denoising maintains the characteristics of the original data.

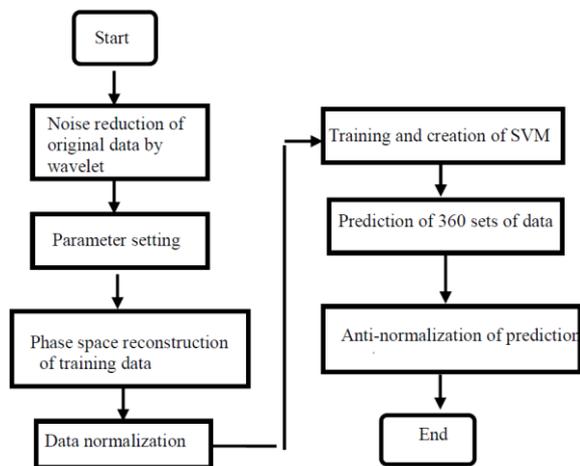
4. Lithium-ion Battery Life Prediction Based on a Phase Space Reconstruction Support Vector Machine

The data processing procedure for the life prediction of Lithium-ion battery is as follows:

- 1) The original test data of Lithium-ion battery is denoised by wavelets to eliminate the influence of temperature and discharge rate change.
- 2) The training data is reconstructed by a phase space to set the phase space dimension $d=5$ and the time delay parameter $\tau = 1$.
- 3) The Mapminmax function in MATLAB is used to normalize the data into the $[0, 1]$ range, in order to remove the influence of the data dimension.
- 4) The support vector machine (SVM) model is created to set the SVM parameters, here to make $c=1, g=0.1$.

- 5) A part of the test data from the lithium-ion battery capacity is used to train and test the model. Here, the training and testing data for the 1060 groups of data have been measured in the first 700 groups, which accounted for 70% and 30% respectively.
- 6) The capacity of the lithium-ion battery is predicted by the prediction model and the remaining 360 sets of data.

The flow chart of data processing method is shown in Figure 8. The predicted result is shown in Figure 9.



Phase space reconstruction of training data

Figure 8: The flow chart of data processing method in the life

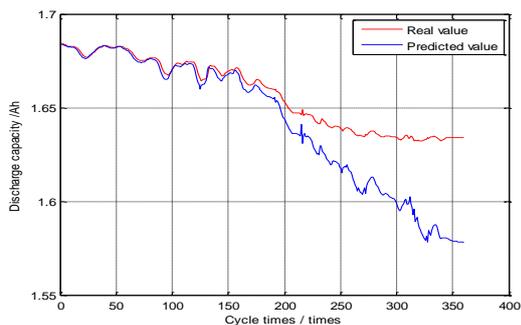


Figure 9: The results of life prediction for Lithium-ion Battery

As shown in Figure 9, the red line indicates the true value, and the blue line indicates the predictive value. The fitting degree of the true value and the predictive value is very high before 150 cycles. With the increase of the sample values, the true value and the predictive value begin to appear obvious deviation after the 150 cycles. The average relative error between the predictive value and the real value of the lithium-ion battery is shown in Table 3.

Table 3: The average relative error between the predicted value and the real value of the lithium-ion battery

Cycle times / times	0-90	91-180	181-270	271-360
The average relative error /%	0.0386	0.1361	0.9210	2.6121

From Table 3, we can see that in the period of 0 to 90 cycles, the average relative error is the smallest, and the average relative error is the maximum in the period of 271 to 360 cycles.

The relative error between the predicted value and the true value of the lithium-ion battery life is shown in Figure 10. The minimum relative error, forward and reverse maximum relative error between the predicted value and the true value is shown in Table 4.

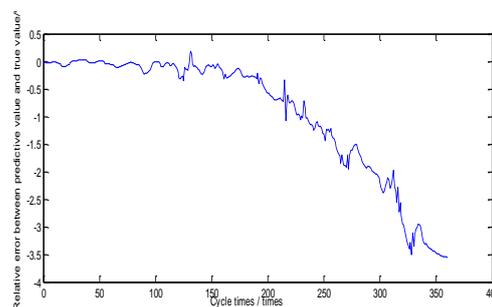


Figure 10: The relative error between the predicted value and the real value of the lithium-ion battery life

Table 4: The relative error between the predicted value and the real value of the lithium-ion battery life

Minimum relative error /%	Reverse maximum relative error /%	Forward maximum relative error /%
0.0018	-3.5538	0.2185

As shown in figure 10, we can see that the relative error between the predicted value and the true value fluctuates between -0.25% and +0.25% before 150 cycles, which shows that the relative error is small. The relative error between the predicted value and the true value is in the range of -0.5% to -3.6% after 150 cycles, which shows that relative error fluctuation is obvious. Overall, the relative error between the predicted value and the true value ranges from -3.6% to +0.25%, so the prediction accuracy is higher.

As shown in table 4, the minimum relative error between the predicted value and the true value of the lithium-ion battery life is 0.0018%, the forward maximum relative error is 0.2185%, and the reverse maximum relative error is -3.5538%.

5. Conclusion

The lithium-ion batteries have been widely used in various fields due to their high energy density and long service life, but their safety problems are the main reasons for the development. The prediction of remaining life of a lithium-ion battery has become an important research direction. The method of a phase space reconstruction support vector machine is proposed to predict the remaining life of a lithium-ion battery, which solves the problem of single input variables, and improves the prediction accuracy. The method is also applicable to the field of power system load forecasting, state prediction of mechanical equipment and so on.

The following conclusions are drawn as follows:

- 1) The original test data of the capacity of lithium-ion batteries was denoised by the wavelet to eliminate the influence of temperature and discharge rate, and improve the accuracy of prediction results.
- 2) The phase space reconstruction of the data after denoising was carried out, setting $d=5$, $\tau=1$.
- 3) A part of the test data from the lithium-ion battery capacity was used to train and test the model. Here, the training and testing data for the 1060 groups of data had been measured in the first 700 groups, which accounted for 70% and 30%, respectively. The capacity of the lithium-ion battery was predicted by using the prediction model and the remaining 360 sets of data. The relative error between the predicted value and the true value of lithium-ion battery capacity is between -3.6% and +0.25%, which indicates that the method of a phase space reconstruction support vector machine is highly accurate. This method is suitable for predicting the remaining life of a lithium-ion battery.

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