A Novel Hybrid Algorithm of Probabilistic Load Flow for Distribution Grids

[1]Hui-Juan Zhang, [1]Yao Feng, [2]Wei-Peng Zhang and [3,]*Li-Wen Wang

Abstract

Involving Latin hypercube sampling and a cumulant method, a novel hybrid method is presented herein to analyze the probabilistic load flow of distribution grids. As the first step, a linear model is established to describe the behaviors of a probabilistic load flow. Each comulant of input random variables is evaluated in the linear model using Latin hypercube sampling and the conventional numerical method. Then the cumulants of output random variables are found using a Gram-Charlier expansion, which is used to obtain the cumulative distribution functions (CDFs) of the output random variables. The outperformance of this novel algorithm over a conventional counterpart is demonstrated in a case study. This proposal shows a good performance agreement with the counterpart, which provides a higher accuracy, and can be viewed as a reliable means for electrical dispatch analysis.

Keywords: Distribution Grids, Probabilistic load flow; Cumulant method; Latin hypercube sampling

1. Introduction

A microgrid refers to a small-scale power supply network with cogeneration at low voltage levels, which supplies electricity or heat load to a small community, such as residential areas, suburbs, industrial parks or other public communities [1]. A microgrid appears as an integration between distributed power generation systems and loads at the voltage level provided by a distribution network [2]. In most cases, generators and energy sources integrated on a microgrid are renewable and alternative, which generate power at the above-stated voltage level [3].

Micropower usually consists of a cogeneration system, a wind energy conversion system, a photovoltaic system, fuel cells, a micro-hydropower, other renewable energy sources and energy storage devices [4]. A number of micro powers in distribution grids, such as wind turbines, fuel cells and solar cells, experience a certain degree of uncertainty in the output power provided. This uncertainty would demonstrate an impact on the operation analysis of distribution grids, such as voltage stability, power quality disturbances and the power quality of sensitive consumers as well as the distribution of the entire distribution grids [5]. It brings about a huge challenge to the operation and control of distribution grids. Therefore, the issue of PLF has been long handled as the basics of the characterization and stability analysis of distribution grids.

It is rather difficult to accurately simulate the performance of wind and solar power sources because they are susceptible to the ambient temperature, the weather and even climate. The instability of active power output is expected to degrade the stability of a system containing micropower, meaning that the voltage quality cannot be well maintained. The stochastic nature of distribution grids cannot be modeled in a deterministic way, while it can be instead well described by a stochastic model, such as the Monte-Carlo simulation method, the cumulant method, the point estimate method and the convolution method [6, 7].

The point estimate method is proposed in a number of studies [8, 9] as a way to deal with power flow in an unbalanced distribution network involving wind and solar power sources. It is developed particularly to effectively resolve the uncertainty of active power output provided by the wind and solar energy sources. However, it requires a huge computation load, resulting in a low accuracy, and is unable to handle discrete random variables.

An extended Latin hypercube sampling technique [10] is developed as an improved version of existing Latin hypercube sampling approaches, and is successfully applied to PLF analysis. Sampling can be accurately performed on a continuous PDF, but unfortunately not on a discrete one.

In recent times, a novel and high performance algorithm has been developed with the aim of improving the computational efficiency of PLF analysis. This is done by using cumulants and a Gram–Charlier expansion [11, 12] to calculate the PDF and the CDF of output random variables. Due to the uncertainty experienced in a network configuration, a combined form of a compensation method and the total probability theorem is presented in [13] to solve the random variation problem in a network configuration, and the probability distributions of transmission line flows were

*Corresponding Author: Li-Wen Wang
(E-mail: 1291985507@qq.com)
[1] Teaching Center of Electrical and electronic, Hebei University of Technology Tianjin, 300130, China
[2] Tianjin Chengxi District Power Supply Company, Tianjin 300110, China
[3] Tianjin Shengli Intellectual Property Agency Co., LTD, Tianjin 300384, China

evaluated in the same way as in [11, 12]. Nonetheless, it is rather difficult to calculate the cumulants of some random variables using a conventional numerical method.

In an attempt to suppress the level of load flow uncertainty, cumulants and a Gram–Charlier expansion are integrated as a novel way to handle the PLF of a power system containing a large-scale wind power [14], and as an effective way to calculate the probability distributions of output random variables. This method, as opposed to a Monte Carlo simulation, is able to approximate the probability distribution of transmission line flows accurately.

Latin hypercube sampling and a cumulant method are integrated herein in such a way that an analysis on PLF, involving micro power, is made accurate. It is that Latin hypercube sampling is firstly performed to analyze the input random variables of distribution grids, and each cumulant of the input random variables is evaluated using a conventional numerical method [15], involving convolution operations, and a Gram-Charlier expansion. Consequently, the CDFs of the random variables are obtained. The accuracy outperformance of this proposal over a conventional counterpart is validated in an IEEE34 node distribution network [16] at the end of this work.

2. Linear Probabilistic Load Flow Model

The behaviors of power injections [17] and line flows in polar coordinates can be simply formulated as

$$\begin{cases} W = g(X) \\ H = k(X) \end{cases} \tag{1}$$

where W represents the input vector of node power injections, H the output vector of line flows, X the node state variables, g the power injections function, and k the line flows function.
Then Equation 1 is expressed as

$$\begin{cases} X = X_0 + \Delta X \\ W = W_0 + \Delta W \\ H = H_0 + \Delta H \end{cases} \tag{2}$$

where X_0, W_0 and H_0 are the expected values of the node state variables, the power injections and the line flows, respectively, related by $W_0 = g(X_0)$ and $H_0 = k(X_0)$. ΔX, ΔW and ΔH are the variable quantities of the node state variables, the power injections and the line flows, respectively.

As in [18], omitting the third and higher order terms, Taylor series expansion of Equation 2 gives

$$\Delta X = J_0^{-1} \Delta W = S_0 \Delta W \tag{3}$$

$$\Delta H = G_0 S_0 \Delta W = T_0 \Delta W \tag{4}$$

where J_0 denotes the last iteration of load flow Jacobian matrix, S_0 the inverse Jacobian matrix, G_0 a matrix of dimension 2b × 2n (b and n stand for the numbers of branches and nodes, respectively), and $T_0 = G_0 \times S_0$.

Under normal conditions, the load flow problem is solved for the node state variables, the expectations of line flows and Jacobian matrices using the Newton – Raphson iteration method [19].

Due to the additivity of cumulants, convolution operations are replaced with additions, and each cumulant of the input random variables can be found using Latin hypercube sampling and conventional numerical methods. Equations (3) and (4) give the cumulants of the node state variables and line flows, and consequently the CDFs of the output random variables are obtained using a Gram-Charlier expansion.

3. Latin Hypercube Sampling Method

According to the characteristics of input random variables, cumulants of random variables with continuous PDFs are found using Latin hypercube sampling method.

The power injections of wind turbines, solar cells and partial load demand are modeled as random variables with continuous PDFs. Each cumulant of the input random variables is evaluated, using Latin hypercube sampling method which provides a uniform distribution of sampling points and thus converges rapidly. Latin hypercube sampling is performed based on an inverse function. Suppose that there are a total of m input random variables $x_k \in \{x_1, x_2, \ldots, x_m\}$, each of which is sampled N times; the CDF of the input random variable X_k is represented as $Y_k = F_k(X_k)$, and the interval [0, 1] on the vertical axis of CDF is divided into N equal-sized subintervals, that is to say, the size of each subinterval 1 / N . Consequently, a random number is chosen from each subinterval as a sample value of the input random variables [20].

4. PLF Calculation

4.1 Deterministic Power Flow

The expectations of the node state variables X_0 and the expectations of line flows H_0 and Jacobian matrices J_0 are found using the Newton- Raphson iteration method, and then S_0 and T_0 are evaluated.

4.2. Cumulant Calculation

The power injection outputs of wind turbines, solar cells and partial load demand are modeled as the random variables obeying a Weibull, a Beta and a normal distribution [21, 22], respectively. Given the PDFs of the random variables, a sample set, symbolized as $\{x_1, x_2, \ldots, x_N\}$, is built using the Latin hypercube sampling method, and the origin moment [23] at each order is given as

$$\chi_r = \frac{1}{N}\sum_{i=1}^{N} x_i^r, \quad r = 1,2,\ldots \tag{5}$$

The cumulant of the power injections output random variables with a continuous PDF [23] at each order is expressed as.

$$\begin{cases} k_1 = \chi_1 \\ k_{r+1} = \chi_{r+1} - \sum_{j=1}^{r} C_r^j \chi_j k_{r-j-1} \end{cases} \tag{6}$$

Input random variables with a discrete PDF can be calculated using a conventional numerical method.

Operations of a fuel cell can be categorized into normal and failed states, and the active power output provided is modeled as a random variable with a discrete PDF in the failed state. Given the number of fuel cells, the failure rate, the unit capacity, the expectations of output power, and the probabilities of active and reactive load with discrete PDFs can be found. According to the conversion relationship between the central moment and cumulants, the cumulant at each order can be evaluated using a conventional numerical method.

The input random variable ΔW_i at node i is composed of a random variable of the generator output power ΔW_{Gi} and the load demand ΔW_{Li} [24], expressed as

$$\Delta W_i = \Delta W_{Gi} \oplus \Delta W_{Li} \tag{7}$$

where $\oplus$ denotes the convolution operator.

For the reason that all the power injections are independently calculated, the distribution functions of the output random variables can be obtained using an algebraic operation on cumulants and a series expansion rather than a convolution operation. The kth-order cumulant $\Delta W_i^{(k)}$ of the total power injections at node i can be expressed as

$$\Delta W_i^{(k)} = \Delta W_{Gi}^{(k)} + \Delta W_{Li}^{(k)} \tag{8}$$

where $\Delta W_{Gi}^{(k)}$ and $\Delta W_{Li}^{(k)}$ represent the kth-order cumulants of the generator output power and the load demand at node i, respectively.

The kth-order cumulants $\Delta X^{(k)}$ and $\Delta H^{(k)}$ of the output random variables can be respectively written as

$$\begin{cases} \Delta X^{(k)} = S_0{}^{(k)} \bullet \Delta W^{(k)} \\ \Delta H^{(k)} = T_0^{(k)} \bullet \Delta W^{(k)} \end{cases} \tag{9}$$

where $\Delta W^{(k)}$ symbolizes the kth-order cumulants of W, and $S_0{}^{(k)}$ and $T_0{}^{(k)}$ are composed of the kth power of elements of the matrices S_0 and T_0, respectively.

4.3. Calculation of Output Random Variables

Three series expansions are involved herein when dealing with the probability distributions of the output random variables, that is, Gram-Charlier, Edgeworth and Cornish-Fisher series expansions [25]. Gram-Charlier series expansion is adopted in this work due to its accuracy outperformance over the other two for non-normally distributed variables, and the CDFs of the output random variables can be found accordingly.

4.4. Computation Steps

Step-by-step PLF calculation by a sampling and a cumulant method for distribution grids is illustrated as a flow chart in Fig. 1, and is summarized as follows.

1) Read basic data, such as the correlation data required by deterministic load flow, the distribution functions of input random variables and a correlation matrix, etc.
2) Run Newton-Raphson load flow at the working point with the basic data. Then the values of the output random variables X_0, H_0, the Jacobian matrix J_0 and the inverse Jacobian matrix S_0 at the working point are all calculated.

3) According to the distribution function of the input random variables, choose an appropriate method between Latin hypercube sampling and a conventional numerical method to calculate its cumulants. If the input random variables have discrete (continuous) PDFs, then the 8th-order cumulants of the input random variables are calculated by the conventional numerical (Latin hypercube sampling) method.
4) The kth-order cumulants $\Delta W_i^{(k)}$ of the total power injections at node *i* can be calculated by Equation 8, and the kth-order cumulants $\Delta X^{(k)}$ and $\Delta H^{(k)}$ of the output random variables can be calculated by Equation 9, (the 8th-order cumulants meet the accuracy demands).
5) Estimate the probability distributions of the output random variables by a Gram-Charlier expansion with the cumulants obtained in step 4.

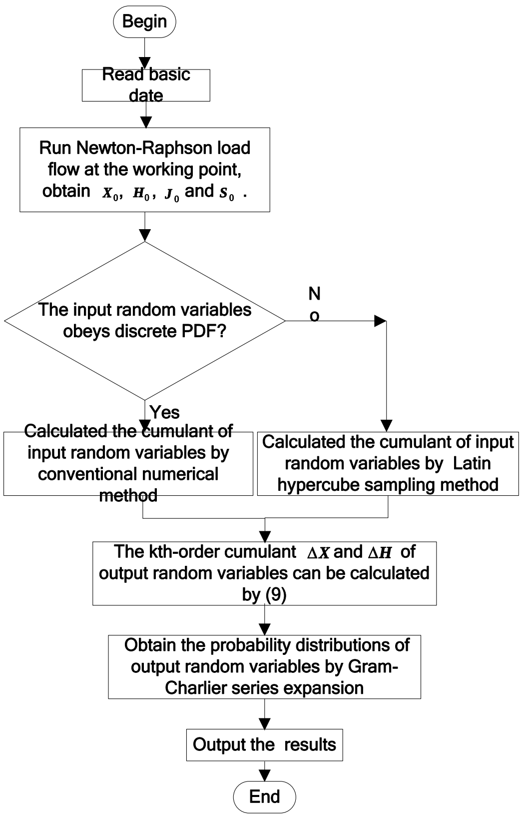

Figure 1: Flow Chart of a PDF Calculation in this work

5. Case Study

An IEEE 34 node distribution network [16] employed for a case study provides a number of micro-powers, as illustrated in Fig. 2, a base voltage of V_b 24.9 kV, a reference voltage of 1.03 p.u. = 25.647 kV in the root node in all cases V_{ref} and a base apparent power S_b of 1M VA. The accuracy of this proposal is validated with the results calculated by the conventional cumulant method as a benchmark. Simulation model and parameters are detailed as follows.

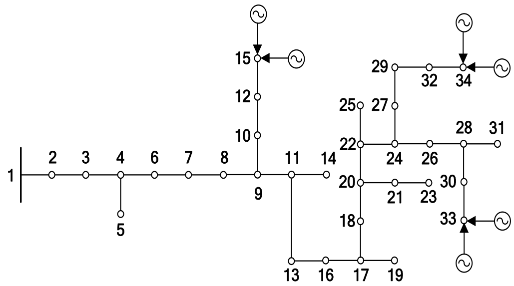

Figure 2: IEEE34 node distribution network

As illustrated in Fig. 2, two wind turbines are connected to node 15, and 10 fuel cells and a wind turbine are connected to node 33; a solar cell array and a wind generator are connected to node 34.

The load demand obeys a normal distribution, and all the input random variables are independent. The parameters of micropowers are given as follows

Wind generator: the rated wind speed is 15 m/s. The cut-in speed is 4 m/s; the cut-out speed is 25 m/s; the rated power is 0.2 MW. The PDF of the wind speed at a wind farm is described by the shape parameter (k = 3.97) [21] and the scale parameter (c = 10.7) of a Weibull distribution function.

Fuel cells: the rated power is 0.05 MW; the failure rate is 0.08, and the mean value is 0.04.

Solar cells: the rated power is 0.1 MW; the area of each solar cell is 2.16 m^2; the photoelectric conversion efficiency of a single solar cell is 13.44%; the photovoltaic array has 400 solar cells, and the PDF of the solar power is described by the shape parameter of a beta distribution function [22] (α = 0.449933, β = 9.186967).

For comparison purposes, Table 1 gives the mean values μ and the standard deviation σ of the magnitude of voltage, simulated using MATLAB by the presented algorithm and the conventional counterpart, at specific nodes of the network in Fig. 2, where μ and σ of the phase angle are listed in Table 2.

Table 1: Mean value and standard deviation of the magnitude of the voltage at specific nodes.

node	Voltage magnitude /p.u.			
	The conventional method		The proposed method	
	μ	σ	μ	σ
7	0.998858	0.003632	0.999815	0.003791
12	1.001231	0.006663	1.002948	0.007477
14	0.999702	0.006804	1.001724	0.00788
15	1.001231	0.006663	1.001305	0.007320
19	0.993603	0.008163	0.999157	0.008760
23	0.986281	0.011832	0.984990	0.012315
25	0.986273	0.011841	0.988186	0.011934
30	0.984721	0.011976	0.987922	0.012356
32	0.984893	0.011946	0.98778	0.012443
33	0.984625	0.011985	0.990922	0.012541
34	0.984908	0.011973	0.993363	0.012752

Table 2: Mean value and standard deviation of the phase angle of the voltage at specific nodes.

node	Voltage angle /p.u.			
	The conventional method		The proposed method	
	μ	σ	μ	σ
7	0.773133	0.074062	0.771923	0.074902
12	0.742043	0.076001	0.739503	0.076491
14	0.871032	0.082418	0.868832	0.082568
15	0.740043	0.076011	0.737153	0.076851
19	1.061480	0.098338	1.059660	0.098568
23	1.399420	0.126940	1.397790	0.127360
25	1.443948	0.130222	1.442288	0.130612
30	1.497486	0.134657	1.495556	0.134777
32	1.539012	0.137623	1.538412	0.139073
33	1.497486	0.134657	1.496146	0.135367
34	1.540811	0.137766	1.539541	0.138546

The performance of this proposal is rated in terms of the following aspects.

1) The CDFs of the magnitude in p.u. of voltage are at specific nodes, and the ARMS of the magnitude of voltage are at all the nodes.
2) The relative error of the magnitude and the phase angle of the voltage are at all the nodes.

Accordingly, the performance of this proposal is firstly assessed in terms of the average root mean square (ARMS) error [26], defined as

$$ARMS = \frac{\sqrt{\sum_{i=1}^{N}(B_i - M_i)^2}}{N} \qquad (10)$$

where B_i and M_i symbolize the CDF values of the output random variables calculated by this proposal and the conventional counterpart at point i, respectively, and N represents the number of the total points.

The performance appraisal is as well made in terms of the relative error (ε) [27], defined as

$$\varepsilon = \frac{|C_M - C_B|}{|C_M|} \times 100\% \qquad (11)$$

where C_M and C_B are the values of output random variables calculated by this proposal and the conventional counterpart, respectively.

Figs. 3–6 illustrate a comparison between the CDFs obtained by this proposal and the conventional counterpart for the magnitude of the voltage at nodes 7 , 15 , 19 and 34, respectively. Fig. 7 illustrates the ARMS distribution, defined in Equation 10, for the magnitude of the voltage over all the nodes, and indicates an average of 1.0897% and a maximum of 3.3340% in an IEEE 34 node distribution network. It is hence evident that this proposal results in a low deviation and a high accuracy when describing the probability distribution for the magnitude of the node voltage.

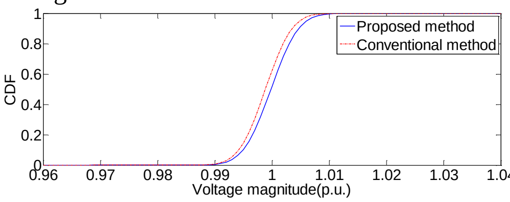

Figure 3: CDF comparison for the magnitude of the voltage at node 7

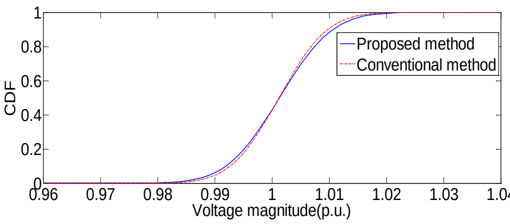

Figure 4: CDF comparison for the magnitude of the voltage at node 15

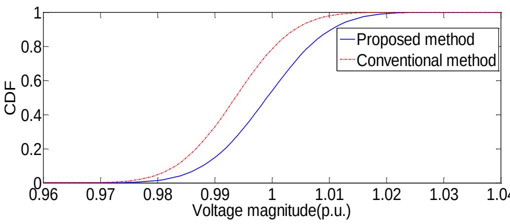

Figure 5: CDF comparison for the magnitude of the voltage at node 19

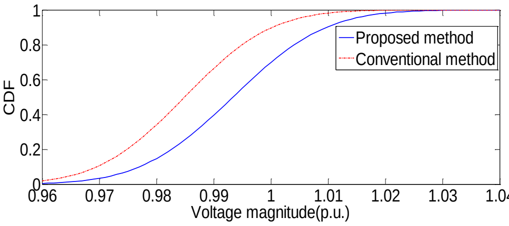

Figure 6: CDF comparison for the magnitude of the voltage at node 34

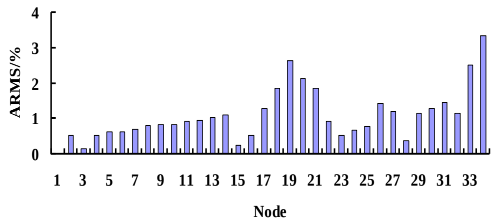

Figure 7: ARMS distribution for the magnitude of the node voltage

Illustrated in Figs. 8–9, there are the elative error, defined in (11), of mean value and standard deviation of the magnitude of the voltage, respectively, and in Figs. 10–11 there are those of the phase angle.

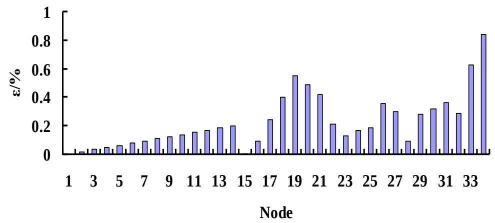

Figure 8: Relative error of the mean value of the magnitude of the node voltage

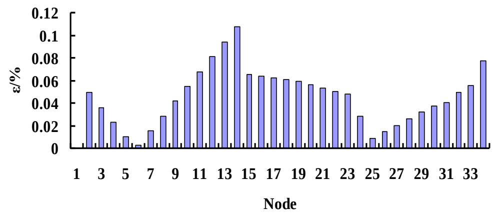

Figure 9: Relative error of the standard deviation of the magnitude of the node voltage

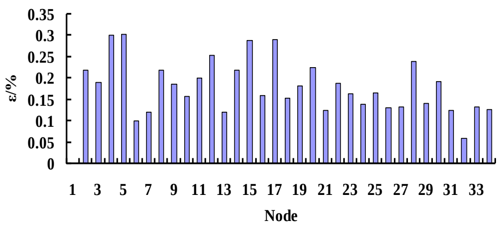

Figure 10: Relative error of the mean value of the phase angle of the node voltage

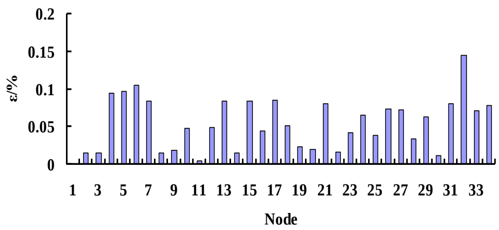

Figure 11: Relative error of the standard deviation of the phase angle of the node voltage

A close observation reveals a maximum relative error of 0.8455% in Fig. 8, 0.1076% in Fig. 9, 0.3020% in Fig. 10 and 0.1450% in Fig. 11. In short, this proposal is found to outperform the conventional counterpart in terms of accuracy, and is as well able to accurately describe the probability distributions of the output random variables.

6. Conclusion

This paper presents a novel algorithm, involving a combined use of Latin hypercube sampling and a cumulant method to evaluate the probabilistic load flow for distribution grids. Simulations are conducted on an IEEE 34 node distribution network for the performance comparison between this proposal and a conventional cumulant counterpart. It gives a good performance agreement with the counterpart, and provides a superior precision. A sampling process involved can be accelerated since evaluations are partially done in an analytic manner. This work is presented as an efficient way to evaluate cumulants and reduce errors, and is validated as a theoretical means for the electrical dispatch and the weaknesses analysis in a power network.

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Hui-Juan Zhang received the B.S. degree in School of Electrical Engineering in 1984 from Hebei University of Technology, Tianjin, China and the M.S. degree in Control Theory and Control Engineering in 1987 and the Ph.D. degree in Electric Machines and Electric Apparatus in 2000. Her research include probabilistic load flow methods to study grid connected distributed power and power system fault diagnosis.

Yao Feng received the B.S. degree in College of Electronic Information and Automation in 2014 from Tianjin University of Science and Technology, Tianjin, China. He is pursuing the Master degree in the School of Electrical Engineering, Hebei University of Technology, Tianjin. His research include probabilistic load flow methods to study grid connected distributed power.

Wei-Peng Zhang received the B.S. degree in School of Electrical Engineering and Automation in 1998 from Tianjin University, Tianjin, China. He is currently a senior engineer in Tianjin Chengxi District Power Supply Company, and has been engaged in the work of electric power system operation management and distribution network maintenance more than 20 years.

Li-Wen Wang received the B.S. degree in Department of Computer Science and Engineering in 1987 from Xidian University, Xi’an, China. He is currently a senior engineer in Tianjin Shengli Intellectual Property Agency Co., LTD. He is the corresponding author of this paper. E-mail: 1291985507@qq.com